RELIABILITY ANALYSIS AND DESIGN OF PRESTRESSED CONCRETE BEAMS AT DIFFERENT LIMIT STATES

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To

GODDESS MAHAMAYI

CERTIFICATE

This is to certify that the thesis entitled 'Reliability Analysis and Design of Prestressed Concrete Beams at Different Limit States' by R. Ranganathan is a record of work carried out under my supervision and has not been submitted elsewhere for a degree.

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LIST OF SYMBOLS

All symbols are explained when they appear first in the text. Only those symbols which appear more than once are listed here.

A_{ts}	=	Area of steel
A_{tsf}	=	Area of steel required to develop the ultimate compressive strength of overhanging portion of the flange
A_{tsw}	=	$A_{ts} - A_{tsf}$
C_c	=	Compressive force in concrete
D_s	=	Diameter of steel
D_{sm}	=	Mean value of D_s
E_c	=	Young's Modulus of concrete
E_s	=	Young's Modulus of steel
F	=	Failure
$F_{M_{ro}}(M_{ro})$	=	Cumulative distribution function of M_{ro}. Similar definitions for $F_{M_{ru}}(M_{ru})$, $F_{M_e}(M_e)$, $F_{v_d}(v_d)$, $F_{v_u}(v_u)$ etc.
F_g	=	Dead load factor
F_q	=	Live load factor
I	=	Moment of inertia
K	=	Number of critical sections in a failure mode
L_u	=	Unsupported length of the beam
M_e	=	External bending moment
M_{eu}	=	External moment under ultimate load

M_{ew}	=	External moment under working load
M_g	=	Working moment due to dead load
M_q	=	Working moment due to live load
M_r	=	Ultimate resisting moment of the section
M_{rc}	=	Resisting moment of the section at initial crack formation
M_{rcm}	=	Mean value of M_{rc}
M_{ro}	=	Ultimate resisting moment of the over reinforced section
M_{ru}	=	Ultimate resisting moment of the under reinforced section
$\bar{M}_{rom}$	=	Mean value of M_{ro}
M_{rum}	=	Mean value of M_{ru}
M_{rom}, s_{ro}	=	Parameters of M_{ro} following type III external (smallest) distribution
N	=	Number of groups of concrete data
O	=	Over-reinforced
P_t	=	Prestress at transfer
P(O)	=	Probability of the section becoming over-reinforced
P(U)	=	Probability of the section becoming under-reinforced
Q	=	Live load
Q_k	=	Characteristic live load
$\bar{Q}_m$	=	Mean value of live load
Q_m, s_q	=	Parameters of Q following lognormal distribution
R	=	Resistance, Reliability
S	=	Action
S_b	=	Area of the section under uniform compression at limit state of strength at transfer of prestress

T_c	=	Tensile force in concrete
T_s	=	Tensile force in steel
U	=	Under-reinforced
W	=	Load
X_i	=	Observed frequency in the ith interval
Y_i	=	Expected frequency in the ith interval
a	=	Depth of stress block
b'	=	Breadth of web
b_b	=	Breadth of bottom flange
b_t	=	Breadth of top flange
b'_m	=	Mean value of b'. Similar definitions for b_{tm}, b_{bm}
c_g	=	Deflection coefficient for dead load
c_q	=	Deflection coefficient for live load
c_p	=	Deflection coefficient for prestress
d	=	Effective depth
d_m	=	Mean value of d
e_m	=	Acceptable error in the estimate of mean
e_s	=	Acceptable error in the estimate of standard deviation
$f_{M_{ru}}(M_{ru})$	=	Probability density function of M_{ru}. Similar definition for $f_{M_{ro}}(M_{ro})$
h	=	Total depth of the section
h_m	=	Mean value of h
k	=	Coefficient depending upon the probability that the test values are less than the characteristic strength
ℓ	=	Effective length of the beam

m	=	Number of failure modes
n	=	Number of samples, number of load conditions
n_u	=	Number of samples of under-reinforced case
p_c	=	Probability of cracking
p_{cs}	=	Probability of cracking of the beam for n load conditions
p_{cu}	=	Probability of failure of concrete cube
p_d	=	Probability of the beam becoming unserviceable due to excessive deflection constraint
p_{dd}	=	Probability of the beam becoming unserviceable due to excessive downward deflection constraint
p_{ds}	=	Probability of the beam becoming unserviceable for n load conditions due to excessive downward deflection constraint
p_{du}	=	Probability of the beam becoming unserviceable due to excessive upward deflection constraint
p_f	=	Probability of failure
p_{fi}	=	Probability of occurrence of failure mode i
p_{fo}	=	Probability of failure for over-reinforced case
p_{fs}	=	Probability of failure of the beam for m modes and n load conditions
p_{fs_j}	=	Probability of failure of the beam for load j
p_{fu}	=	Probability of failure for under-reinforced case
p_{fAC}	=	Probability of occurrence of failure mode in span AC
$p_{f\|o}$	=	Conditional probability of failure for given event O
$p_{f\|u}$	=	Conditional probability of failure for given event U
p_s	=	Probability of failure of steel

p_t	=	Probability of failure of the beam at limit state of strength at ~~transfer of prestress~~
p_{ts}	=	Probability of failure of the beam for m failure modes at limit state of strength at transfer of prestress
p_{tAC}	=	Probability of occurrence of the failure mode in span AC at limit state of strength at transfer of prestress
r	=	Number of parameters estimated from the data
s	=	Standard deviation
s_c	=	Standard deviation of concrete. Similar definitions for $s_s, s_b{'}, s_{b_t}, s_h, s_k, s_d, s_{t_1}, s_{t_t}, s_\beta, s_\kappa$ etc.
s_{rc}	=	Standard deviation of M_{rc}
s_{ru}	=	Standard deviation of M_{ru}
$\bar{s}_q$	=	Standard deviation of live load
$\bar{s}_{ro}$	=	Standard deviation of M_{ro}
$\bar{s}_{vd}$	=	Standard deviation of v_d
$\bar{s}_{vu}$	=	Standard deviation of v_u
s_β^2	=	Variance of β
s_k^2	=	Variance of k
t	=	't' statistic
t_b	=	Thickness of flange at bottom
t_{bm}	=	Mean value of t_b
t_t	=	Thickness of flange at top
t_{tm}	=	Mean value of t_t
u	=	Deflection

u_{cc} = Deflection due to creep of concrete

u_d = Downward deflection

u_g = Deflection due to dead load

u_p = Deflection due to prestress

u_q = Deflection due to live load

u_u = Upward deflection

v = The ratio (Deflection/span)

v_d = $(u_d/\ \ell)$

$\bar{v}_{dm}$ = Mean value of v_d

v_{dm}, s_{vd} = Parameters of v_d

v_u = (u_u/ℓ)

$\bar{v}_{um}$ = Mean value of v_u

v_{um}, s_{vu} = Parameters of v_u

w_g = uniformly distributed load due to dead load

w_p = uniformly distributed load due to prestress

w_q = uniformly distributed load due to live load

x_b = The distance of centroid of steel from bottom

y_c = Central ordinate of parabolic cable profile

β = The ratio of characteristic strength of concrete to the mean strength

β_m = Mean value of β

χ^2 = Value of chi square

ρ = Correlation coefficient

δ = Coefficient of variation

δ_i = The value of δ for ith group

δ_m = Mean value of δ

δ_q = Coefficient of variation of live load

δ_{rc} = Coefficient of variation of M_{rc}

δ_{ro} = Coefficient of variation of M_{ro}

ε_c = Limiting compressive strain in concrete

ε_{ce} = Strain in concrete due to effective prestress

ε_{ct} = Tensile strain in concrete

ε_s = Strain in steel at the time of failure

ε_{sc} = Strain in steel at initial crack formation

ε_{sp} = Strain in steel due to effective prestress

γ = Material reduction coefficient

γ_c = Material reduction coefficient for concrete

γ_s = Material reduction coefficient for steel

κ = $(0.68\ \sigma_{cu}\ S_b/P_t)$

κ_m = Mean value of κ

λ_s = The ratio of p_{fu} to p_f for PV of Q, σ_{cu} and σ_s

λ_{sd} = The ratio of p_{fu} to p_f for PV of Q, σ_{cu}, σ_s and dimensions of section

ϕ = Cumulative distribution function of a standardized normal random variable

ψ = The coefficient used to calculate allowable deflection

σ_{cu} = Cube strength of concrete at 28 days

σ_{cum} = Mean value of σ_{cu}

σ_{cul}, s_{cl} = Parameters of σ_{cu} following lognormal distribution

σ_r = Modulus of rupture of concrete

σ_s = Ultimate stress in steel

σ_{sm} = Mean value of σ_s

σ_{su} = Stress in steel at the time of failure of the beam

CTD = Cold twisted deformed

CV = Coefficient of variation

HTS = High tensile steel

NA = Neutral axis

PF = Probability of failure

PSC = Prestressed concrete

PV = Probabilistic Variation

BM = Bending moment

c.g.c. = Centre of gravity of concrete

SYNOPSIS

The thesis presents the statistical analysis of field data on compressive strengths of concrete cubes for characteristic concretes of M150, M200, M250 and M350. The statistical analysis indicates that the most of the groups of concrete follow normal distribution with one percent level of significance. The most probable value of the failure of concrete cubes, computed by the method of least squares, is found to be 5.5 percent. This value checks closely with that recommended by CEB-FIP Committee for international code of practice. Statistical analysis is performed for the data on strengths of high tensile steel and cold twisted deformed steel. Load survey carried out on floors in office rooms and statistical analysis of the same is presented. Statistical analysis of the data collected on geometric parameters of a typical concrete section is also included. The strengths of steel and concrete are always treated as random variables; whereas loads and geometric parameters are treated as deterministic in some examples and probabilistic in some other examples.

The reliability analysis of simply supported and continuous prestressed concrete beams designed as per Indian Standard specifications is presented using statistical analysis of random variables and Monte Carlo technique. The reliability analysis of the above beams is done at limit states of (i) strength, (ii) initial crack, (iii) deflection and (iv) strength at transfer of prestress.

The failure of a prestressed concrete section at limit state of strength is divided into two cases - (i) under-reinforced and (ii) over-reinforced. The probability of failure of a section is evaluated based on the above two conditional failure events.

A method for the reliability based design of simply supported and continuous prestressed concrete beams at limit state of strength for a given probability of failure is formulated. This involves solving of an integral equation and finding the mean value of ultimate resisting moment of the section.

A rational study on load factors is made taking into account of the coefficient of variation of the resistance and probability of failure of the beams. Load factors for dead load and live load are suggested for strength and initial crack limit states. Limit state design of a prestressed concrete beam, based on semi-probabilistic approach, is illustrated.

The probability of failure at transfer condition of prestressed concrete beam designed by Indian Standard Code is found to be higher than that at limit states of strength and deflection during working load condition. The probability of cracking of the beams is generally higher than the probability of failure at limit states of strength and deflection.

CHAPTER 1

INTRODUCTION

1.1 General

The probabilistic approach to structural safety in civil engineering has been one of the subjects of interest in the last 15 years. Great strides have been made to accurately compute stresses, deflection, buckling loads etc. If loads and resistances are known exactly, then significant reductions in the factor of safety and improved economical benefits can be achieved. In fact, for many problems, loads and resistances can be described as statistical variables and therefore, the safety of the structure can also be statistical variable. It would appear that improved structural analysis and design techniques should be accompanied by improved methods of predicting structural safety. It has been proposed for sometime that a rational criterion for the safety of the structure is its reliability or probability of survival. Statistical frequency distributions for loads and strengths must be considered in determining the reliability. Freudenthal said (4)* : 'Because the design of a structure embodies uncertain predictions of the structural materials as well as of the expected load patterns and intensities, the concept of probability must form an integral part of any rational

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*Numbers in paranthesis refer to corresponding items in References.

design; any conceivable condition is necessarily associated with a numerical measure of the probability of its occurrence. It is by this measure alone that the structural significance of a specified condition can be evaluated'.

Much work was done on the basic concept of structural safety, in 1950s. In 1960s, most of the contribution was towards the probabilistic analysis and design of structural systems. In 1970, CEB-FIP Committee (45) recommended semiprobabilistic limit state design method in which basic parameters are considered as random and the coefficient related to safety is introduced by using limit state method where increased loads are compared with the relevant resistance of the structure and where effects of service loads are compared with specified values. Currently Ang, Amin, Cornell and Lind (18, 24, 30, 34, 37) have developed reliability design formats where uncertainty is expressed in terms of the coefficient of variation without frequency distributions of individual variates. Literature dealing with reliability analysis and design of structures is reviewed in the present thesis.

1.2 REVIEW OF LITERATURE ON PROBABILISTIC ANALYSIS AND DESIGN

Most of the literature available in English language is reviewed in chronological order.

A comprehensive treatise on the safety of structures and on the factor of safety in particular was presented by Freudenthal

in 1947. The basic concept of structural safety analysis was first outlined by him (1). This problem was later discussed by Asplund (2) and Pugsley (3).

It was shown (4) that the probability of failure, p_f, can be computed as follows.

$$P(R < S) = p_f = \int_0^\infty [1 - F_S(r)] f_R(r)\, dr \tag{1.1a}$$

$$= \int_0^\infty F_R(s)\, f_S(s)\, ds \tag{1.1b}$$

in which $f_R(r)$ and $f_S(s)$ are equal to the derivatives of disribution functions $F_R(r)$ and $F_S(s)$ respectively. R and S represent resistance and action (load) respectively. The p_f can be expressed in terms of a design factor of safety and thus is used to provide for a probabilistic interpretation of the factor of safety.

Julian (5) presented statistical data on the yield strength and ultimate strength of A-7 steel, the yield strength of intermediate grade steel and ultimate compressive strength of concrete.

The paper (6) deals with the methods of providing engineering safety and additional safety necessary for social purposes. The methods considered for providing safety were by specifying (i) a minimum ratio of resistance to maximum design load (ii) a maximum value for probability of failure and (iii) a minimum factor of safety coupled with maximum probability of failure. The concept of structural life in design was discussed.

Milik Ticky and Milos Vorliecek (7) formulated the problem of safety analysis of reinforced concrete framed structures subjected to load from one source and with several collapse mechanisms and load from several sources. Pearson's type III curve was suggested for the ultimate strength of the structure. Ultimate strength of statically determinate and indeterminate structures were compared to find the relation between statistical parameters of their ultimate strengths produced under identical conditions and demonstrated that variability in ultimate strength of redundant structure is lower than that of statically determinate one in all cases. It was also shown how the deformation ability of critical sections can also be taken into account in the study of safety of structures. An approximate method was developed for evaluating the multiple integrals for reliability expression.

The work of the Task Committee on factors of safety since 1965 were summerized by Freudenthal, Garretts and Shinozuka (8). Reliability functions of structures consisting multiple members were derived for the cases when the loads are applied at equal intervals or at prescribed instants and the number of occurrences of the load is governed by a Poisson Law. Numerical examples were also given.

In 1967, Cornell (9) derived expressions for bounds on the reliability of a structure, having number of failure modes and subjected to number of successive loads, for independent, perfectly and imperfectly correlated loads and modal resistances. It was shown

that although the precise determination of the reliability of structural system is a most complicated effort, upper and lower bounds can be readily evaluated.

Moses and Kinser (10,11) presented a paper on optimum sizing of elements for a multielement and multiload conditions of structure with safety defined in terms of allowable probability of failure. Significant weight savings were obtained by computing the statistical correlation between failure modes and developing an ordinary method to find probabilities of failure of a mode conditional upon the survival of the other modes. They also presented a method for evaluating the reliability of a structure (pinned-connected truss) with possible failure modes subject to two load conditions taking strength and load as probabilitic.

Turkstra (12) formulated the choice of safety level in structural design on the basis of minimum expected loss criterion. Bounds on sensitivity to failure probabilities had been established based on the insignificance of initial costs. The collapse or limit failure of a system was examined by Shinozuka and Hanai (13) and it was shown that the exact reliability expression requires the evaluation of the multiple integrals.

One of the important tools in reliability analysis is the hazard function or risk function. In 1968, Ang and Amin (14) established a monotonic property for the hazard function of structures. The paper (15) describes some of the advantages of rational probabilistic

analysis and design concepts with existing deterministic procedures. The design decision process for seismic loading was illustrated using statistical decision theory. A Monte Carlo simulation technique was described by Warner and Kabaila (16) which can be used to determine the cumulative distribution functions of stochastic variables such as ultimate strength and factor of safety. Using this the variability in strength of an exially loaded short reinforced concrete column was investigated and the results were compared with closed form solution.

Procedures were presented by Sexsmith (17) for determining the probability distribution of the safety margin and related quantities for several types of reinforced concrete members. The procedure examines the structure and its components at a factored load (near ultimate) and considers the probability of failure under this loading.

A formulation of structural safety was proposed by Ang and Amin (18) in which the lack of knowledge and information is handled through a judgement factor whereas the statistical variables of information are treated probabilistically.

The paper (19) deals with the selection of probability distributions for wind load, snow load and plastic moment of a section. The plastic collapse failure of a system was investigated. The method of determining probability of failure of a frame by individual failure mechanisms was presented. It was shown that the probability of failure of the frame involves the evaluation of multiple integrals.

Approximate methods were suggested for evaluating the probability of failure of a frame.

The advantages of probabilistic concepts were discussed by Shah (20). He explained a format for the calculation of probability of failure of the structure, using only mean values and standard deviations of resistance and load and assuming probability distribution of (R-S) or (R/L).

Costello and Chu (21) assumed Beta distributions for strengths of steel and concrete and based on these, probabilities of failures of reinforced concrete rectangular beams were presented.

The paper (22) illustrated various difficulties, such as (i) choice of the appropriate probability model, (ii) introduction of subjective elements into the probabilistic structure and (iii) interpretation of probabilistic information in a form which leads to rational decisions, in the application of probabilistic concepts. Benjamin and Lind (23) investigated the approximate probability measures associated with the existing code provisions. Reliability based procedures were illustrated using a set of assigned probabilities. The concept is to relate probability measures to existing deterministic code provisions thereby allowing improvement of design procedures within the deteministic existing code format.

Cornell (24) presented a consistent first order reliability code format based on first and second moments of all stochastic variables.

Yao and Yeh formulated the safety analysis of structural systems which yields a systematic method of counting the failure paths. The reliability of a parallel and redundant system was formulated in this manner (25). Some examples (trusses and frames) of the optimal reliability based design of both weakest link structures and frames for minimum weight were presented by Stevenson and Moses (26). They developed a method of analysis (27) for determining the overall probability of failure of framed structures (suitable for plastic mechanism analysis) whose failure mechanisms is expressed as a linear combination of several structural resistances and loadings. The correlation between failure modes was taken into account. To determine the reliability of a failure mode, recursive integration method and Pearson distribution method were developed.

Allen (28) made a detailed study on estimated probability distributions for the ultimate bending strength and ductility ratio (curvature at ultimate to curvature at yield) using Monte Carlo technique for rectangular concrete sections reinforced in tension only. He concluded that a section under-reinforced according to ACI 318-63 (29) will undergo a brittle compressive failure. The variability of ductility ratio is much higher than the variability of ultimate moment.

Lind (30) presented reliability code formats of arbitrary higher order than first and second moments of stochastic variables and demonstrated the calibration of a partial safety factor format to Cornell's format as well as to the Ang-Amin format.

The paper (31) proposed a distribution function composed of a central gaussian portion and an exponential tail with a single point of discontinuity as a convenient representation of statistical demand and capacity data to evaluate reliability, regardless of any conjecture about the parent distributions. The convolution integral of the probability of failure is evaluated in closed form in terms of simple statistics of the data.

In 1972, Task committee on structural safety reviewed the available literature till December 1971 and compiled a bibliography on structural safety (32). The feasibility of designing concrete beams for minimum cost with a reliability based constraint was demonstrated by S.S. Rao (33). Instead of specifying small values for probability of failure, the statistical constraint was specified as the design is safe when the expected moment capacity of the beam exceeds the design moment by a certain number of standard deviations.

The importance of risk and uncertainty conditions in structural evaluation and design was emphasized and practical methods for risk assessment in terms of failure probability and for development of reliability based design criteria were described and developed by Ang (34). He suggested the systematic analysis of uncertainty through first order statistical analysis and the explicit use of failure probability as a measure of risk and safety.

Fred Webster (35), in 1973, analysed the structural response of a reinforced concrete portal made up of one beam and two

spirally reinforced columns using probability theory. The portal was subjected to an increasing sequence of static test loads and at each load the probabilities of plastic hinge formation were calculated. Possible paths of failure mode were represented in a Markov chain. A probabilistic based structural design method was introduced by Paloheimo Eero and Hannus Matti (36). The reliability is replaced by a deterministic design equation in which all random variables are substituted by appropriate design values which depend upon the mean values and standard deviation of the corresponding variables, the reliability factors that reflect the acquiried levels of reliability and the sensitivity factors that measure the effects to individual random variables upon the total system. Methods of application were also considered.

Models of risk evaluation were developed by Ang and Cornell (37). The distribution function of (R-S) or ℓn (R/S) was considered without the distributions of individual variates. It was emphasized that the distribution of the safety margin (R-S) or ℓn (R/S) is important in the calculation of probability of failure. Design formats for reliability based design were presented. Uncertainty (including uncertainty associated with errors in estimation and perfection in mathematical model) was expressed in terms of the coefficient of variation. This is amenable for systematic evaluation and analysis of uncertainty in structures. Formulation for multiple load cases was also presented. The quantitative analysis of design

uncertainties was illustrated by Ellingward and Ang (38) and was shown, how these uncertainties affect the level of risk. The risk associated with the existing design procedures were evaluated with reference to reinforced concrete. It was found that the percentage of steel and effective depth are the only variables contributing significantly uncertainty to resisting moment. These risks serve as initial basis for the formulation of risk based design. Using the reliability based design method proposed by Ang and Cornell, design of several beams, columns and cold formed steel beam elements were illustrated by Ravindra, Lind and Su (39) and was shown that code parameters such as the safety indices, can be selected to match the safety level of current designs. Only mean values and standard deviations were used to represent the design random variables. The final design criteria were developed on a probability distribution free basis.

The design formats suggested by Ang and Cornell were used by Moses for reliability analysis of structural systems. Moses (40) introduced partial safety factors which are quantities by which the element safety factor should be changed to account for its presence in a structural assemblage so that the overall probability remains roughly equal to what is desired for the element. Examples of solutions for reliability analysis and design of structural systems were presented.

Probabilistic dynamic approach and problems that are encountered in dealing with the random process were discussed by Shinozuka (41). Models that have been used in the structural response

analysis involving winds and waves were presented. Yao James (42) presented a simple reliability based procedure for the fatigue design of structures.

Putcha Chandrasekar and Dayaratnam (43,44) presented the reliability analysis of prestressed concrete simply supported beams designed on the codes of Indian Standards, American Concrete Institute and Recommended Code of Practice by CEB-FIP. Probable strengths of basic materials used in the beam were generated as random variables subject to code specifications. The probability of failure was calculated based on the governing equation of the ultimate strength of the section fixed by deterministic analysis and normal distribution was assumed for the strength and resistance of the section. The design of prestressed concrete beam for a given reliability was formulated as a polynomial equation. Optimization of prestressed concrete beams subject to the constraint of probability of failure besides the usual constraints was also done.

1.3 STATEMENT OF THE PROBLEM

The object of the present investigation is as follows :

(a) Statistical analysis of the available data on strengths of materials, loads and geometric properties of the section;

(b) Presentation of a method of reliability analysis of prestressed concrete sections at limit state of strength;

(c) Probability failure analysis of prestressed concrete simply supported and continuous beams designed as per the existing

I.S. Code (46) at limit states of (i) strength at design load, (ii) crack initiation, (iii) deflection and (iv) strength at transfer of prestress;

(d) Design of simply supported and continuous prestressed concrete beams for given probability of failure at limit state of strength;

(e) Study of the variation of load factors with coefficient of variation of resistance of the section and probability of failure and to suggest the values of load factors.

The thesis presents the statistical analysis of field data on compressive strength of concrete cubes for characteristic concretes of M150, M200, M250 and M350 and the most probable value of the failure of concrete cubes is computed using the method of least squares. Statistical analysis has been performed for the data on strengths of high tensile steel and cold twisted deformed steel. Load survey carried out on floors in office rooms and statistical analysis of the same is presented. Statistical analysis of the data collected on geometric properties of a typical concrete section is also included.

A general method of analysis of probability of failure of a prestressed concrete section at limit state of strength is presented. Monte Carlo technique is used to study the reliability of sections at limit states of (i) strength, (ii) initial crack, (iii) deflection and (iv) strength at transfer of prestress. Reliability analysis of typical simply supported and continuous prestressed concrete beams

for deterministic and probabilistic loads at different limit states are presented considering probabilistic variations of (i) strengths of concrete and steel and (ii) strengths of concrete and steel and geometric properties of the section. A method of design of simply supported and continuous prestressed concrete beams for given reliability at limit state of strength is formulated and examples are given. Variation of load factors with the coefficient of variation of resistance and probability of failure are studied and the values of load factors for dead load and live load are suggested based on the results of the reliability analysis of prestressed concrete beams. Semi-probabilistic limit state design of prestressed concrete beam is illustrated. Conclusions drawn based on the discussions of the results are presented at the end.

The main purpose of the thesis has been the demonstration of possibilities furnished by the Monte Carlo simulation method. Since the data on random behaviour of various quantities are currently not available in sufficient amount certain simplifications has to be accepted in the soluti They mainly concern the mutual dependence of some random quantities and the deterministic models of limit states. The general approach presented in the thesis can easily be developed after sufficient data are available.

CHAPTER 2

STATISTICAL ANALYSIS OF STRENGTHS OF CONCRETE AND STEEL, LOAD AND DIMENSIONS OF A SECTION

2.1 INTRODUCTION

Acceptable strengths of materials and tolerances and characteristic loads are specified by the codes of practice. A rational approach to the specifications on structural safety needs to be associated with the field data and statistical analysis. A probability analysis and design requires a knowledge of the statistical nature of strengths of materials, loads and the geometry of invidual sections.

The collection and statistical analysis of field data on strengths of concretes used in different projects is first presented*. The frequency distribution of the samples and the probability of failure (PF) of a specified concrete are computed and presented. A characteristic coefficient, which ensures a preassigned reliability of the test results less than the characteristic strength, is obtained by the method of least squares. The work also investigates the relation between characteristic and mean value of the strength of concrete used in some parts of India and compares it with that suggested in

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* This work is reported in reference number (62).

CEB-FIP recommendations. The statistical analysis of data on strengths of high tensile steel (HTS) and cold twisted deformed (CTD) steel is presented with the probability distributions of the same. The survey carried out on floor loads in office building at Indian Institute of Technology, Kanpur, is also given and the frequency distribution of the load is specified. Lastly, a study of the variations in geometric properties of a typical prestressed concrete section is included and the statistical analysis of the data is presented.

2.2 STATISTICAL ANALYSIS OF STRENGTH OF CONCRETES

2.2.1 Collection of Field Data

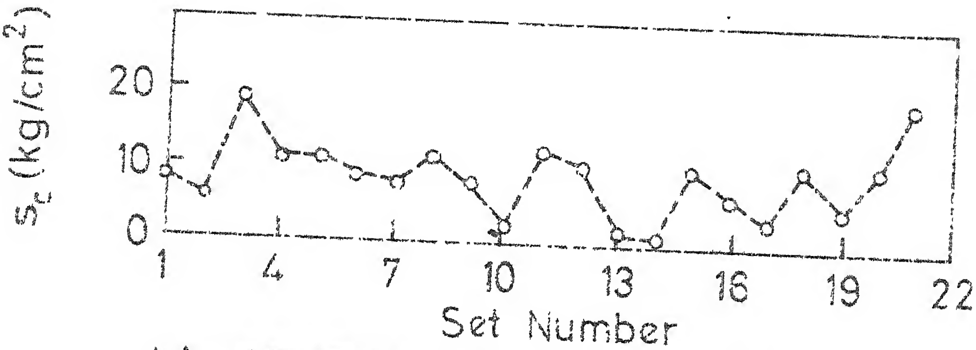
Structural Engineering Laboratory at the Indian Institute of Technology, Kanpur, has been associated with testing of variety of materials used in the construction industry. During the last ten years a large number of samples of cement, concrete and steel have been tested in the laboratory. 150 mm cubes were prepared as per Indian Standard Specifications (47) and supplied either by the builders or by the project supervisors to the laboratory. The projects include small and large scale office, residential and workshop type buildings, bridges and foundation structures. This thesis presents the strength of the cubes after 28 days of water curing.Only those tests with a minimum number of 35 specimens have been considered for the analysis. The specimens of a given project

were supplied in sets of three to six at random. Therefore, all the cubes of a given project were tested as and when received and not in a continuous series. The cubes were tested as per the Indian Standard Specifications (47). The characteristic strength of the concretes which are also called as the specified strengths of the concretes consist of M150 and M200 based on the nominal mix specifications and M250 and M350 proportioned by a design method.

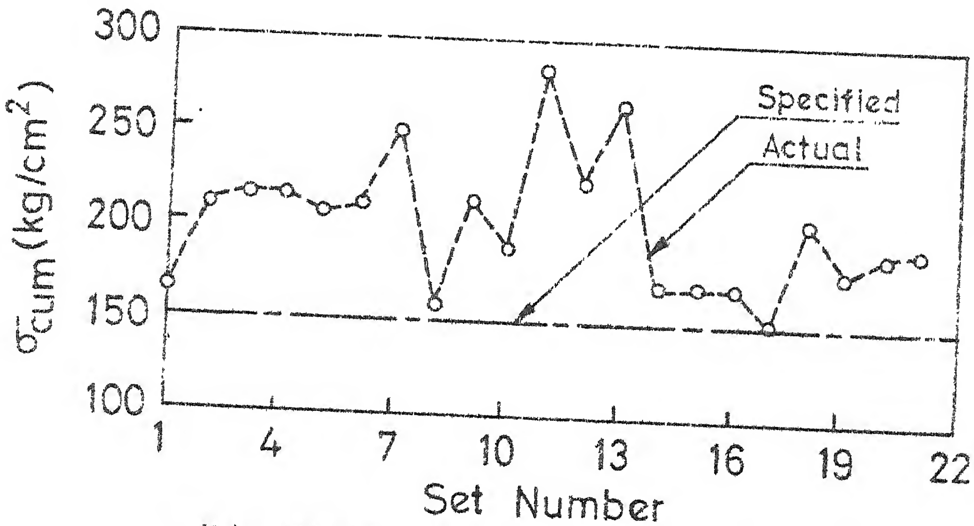
2.2.2 Histograms and Statistical Analysis

The total number of samples of a particular concrete mix (say M150) are divided into project groups. Each group is separated into a number of sets as supplied to the laboratory. Each set consists of usually three samples and sometimes four or six. The mean value and the standard deviation of the strength of concrete of each set is computed and typical variations are given in Figs. 2.1 to 2.4. The figures indicate the random variation of the strength of the concrete. Histograms for three typical groups are shown in Figs. 2.5 to 2.7 indicating the frequency distribution of the strengths of the samples. The mean value and the standard deviation of the strengths belonging to the three groups are given in these figures.

All the samples belonging to a class of concrete are combined and the histogram for such a concrete is drawn.



(a) VARIATION OF STANDARD DEVIATION



(b) VARIATION OF MEAN VALUE

FIG.2.1 VARIATION OF STRENGTH OF M150 CONCRETE FOR A TYPICAL GROUP

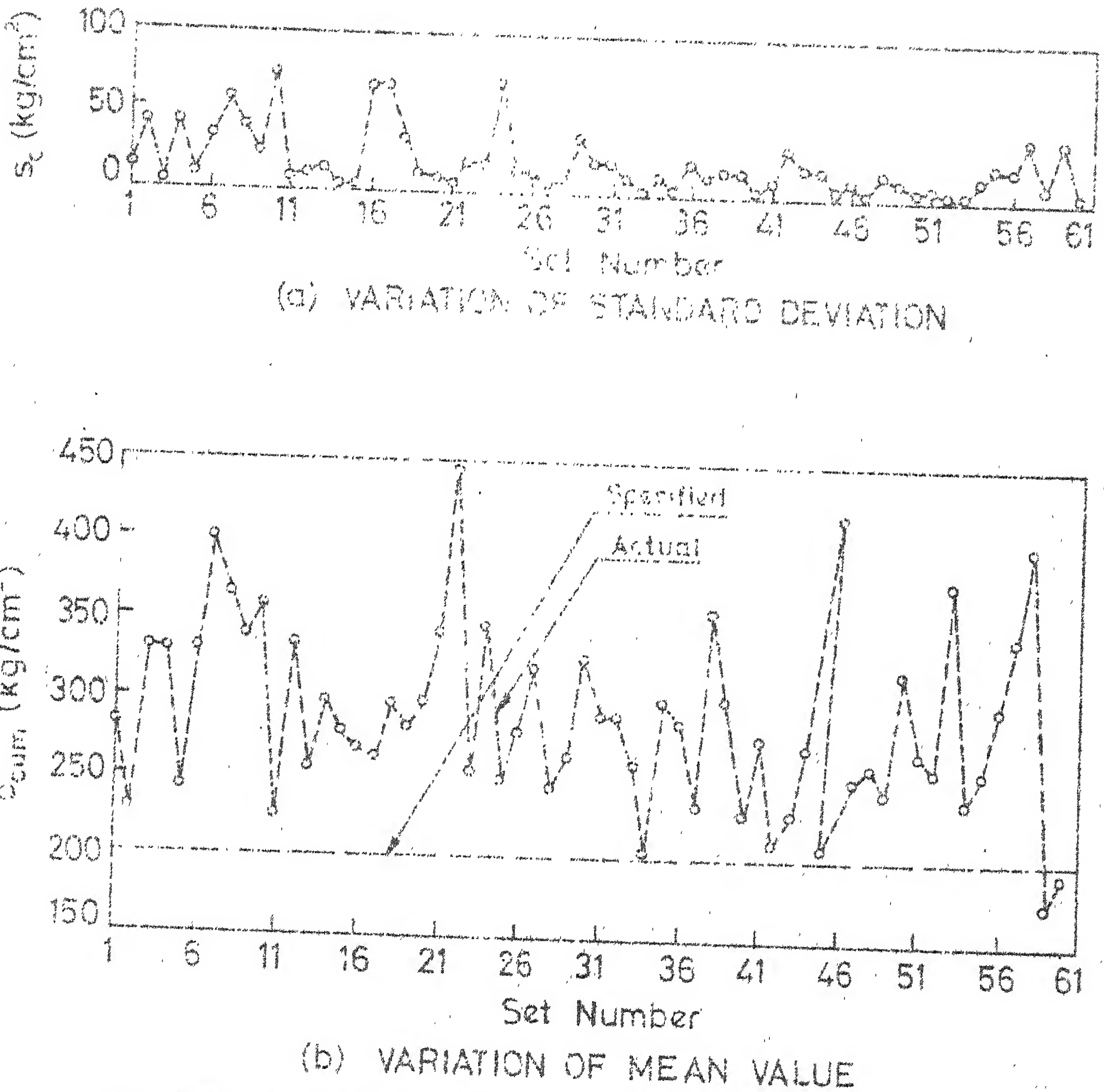
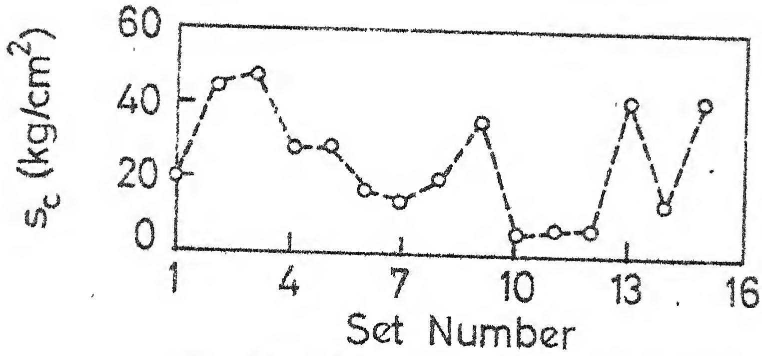
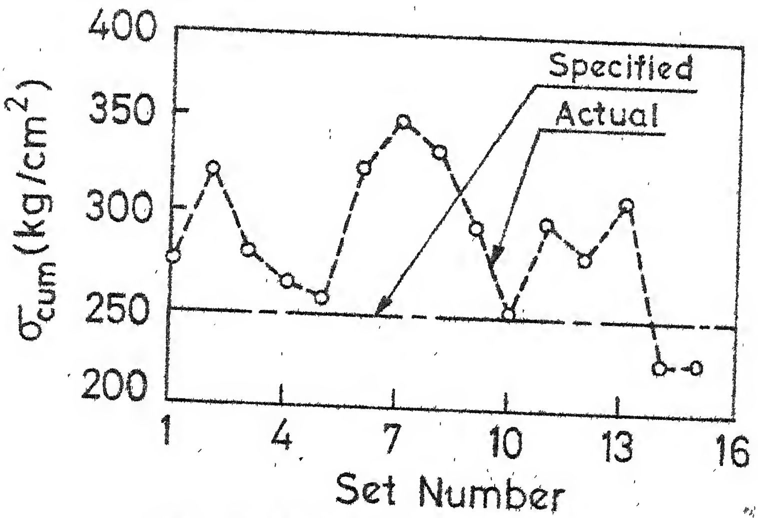


FIG.2.2 VARIATION OF STRENGTH OF M200 CONCRETE FOR A TYPICAL GROUP

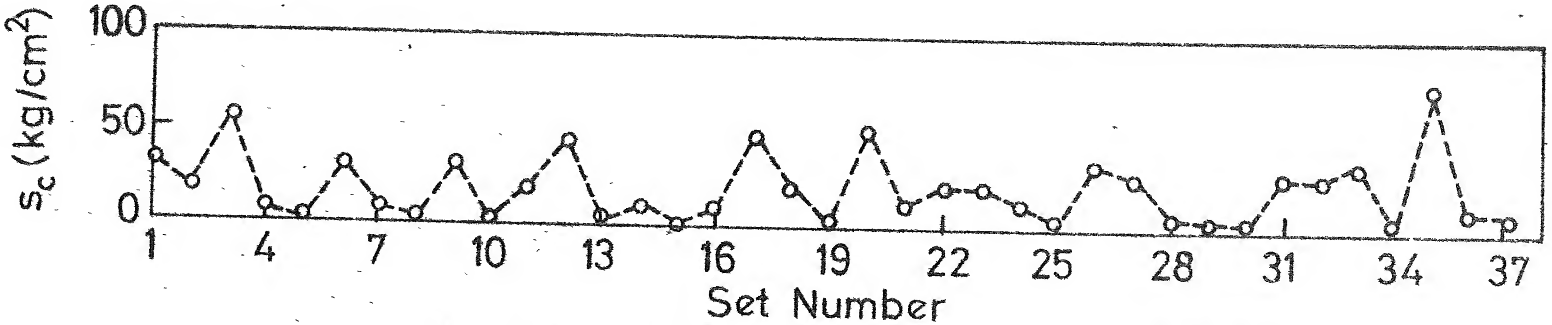


(a) VARIATION OF STANDARD DEVIATION

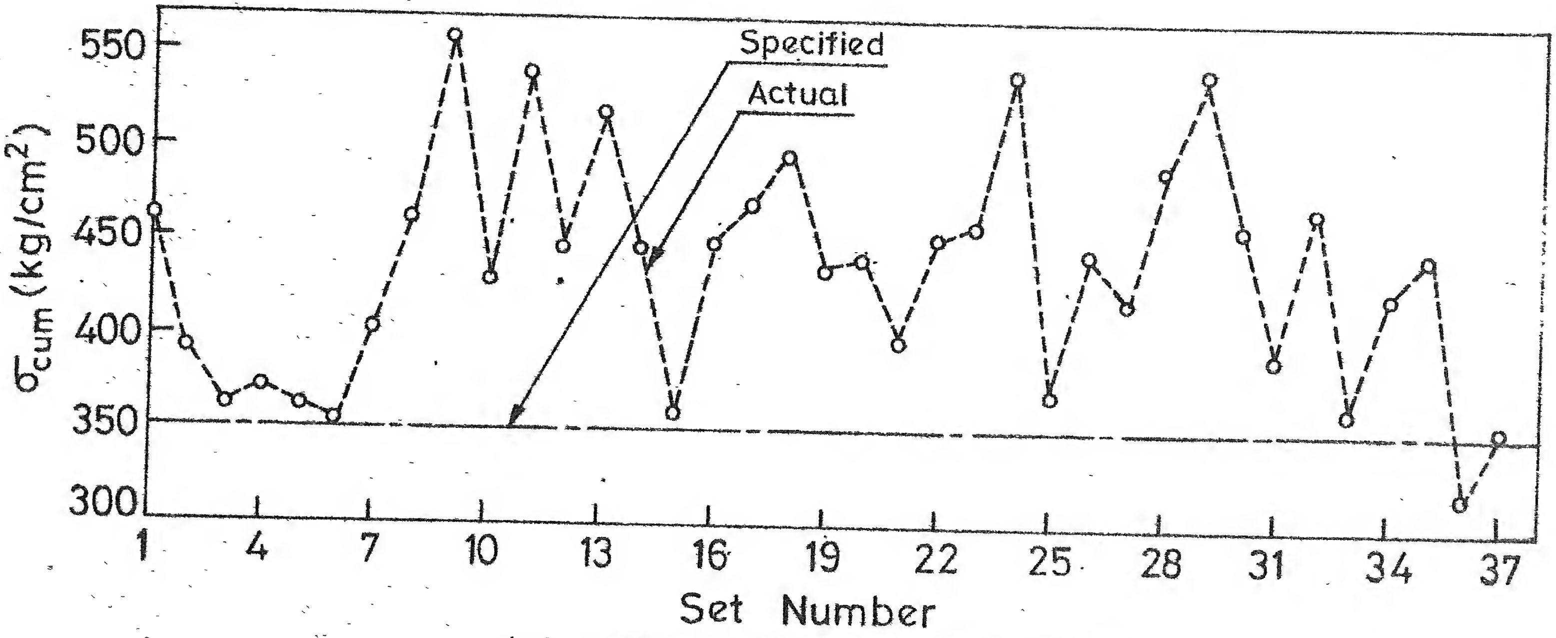


(b) VARIATION OF MEAN VALUE

FIG.2.3 VARIATION OF STRENGTH OF M250 CONCRETE FOR A TYPICAL GROUP.



(a) VARIATION OF STANDARD DEVIATION



(b) VARIATION OF MEAN VALUE

FIG.2.4 VARIATION OF STRENGTH OF M350 CONCRETE FOR A TYPICAL GROUP

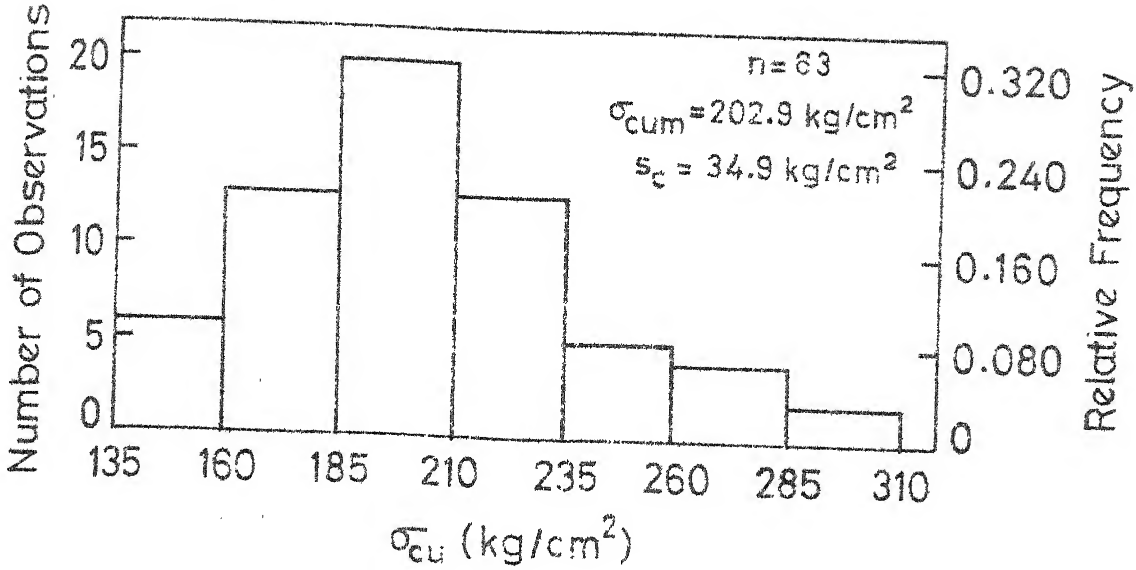


FIG.2.5 HISTOGRAM OF M150 CONCRETE FOR A TYPICAL GROUP

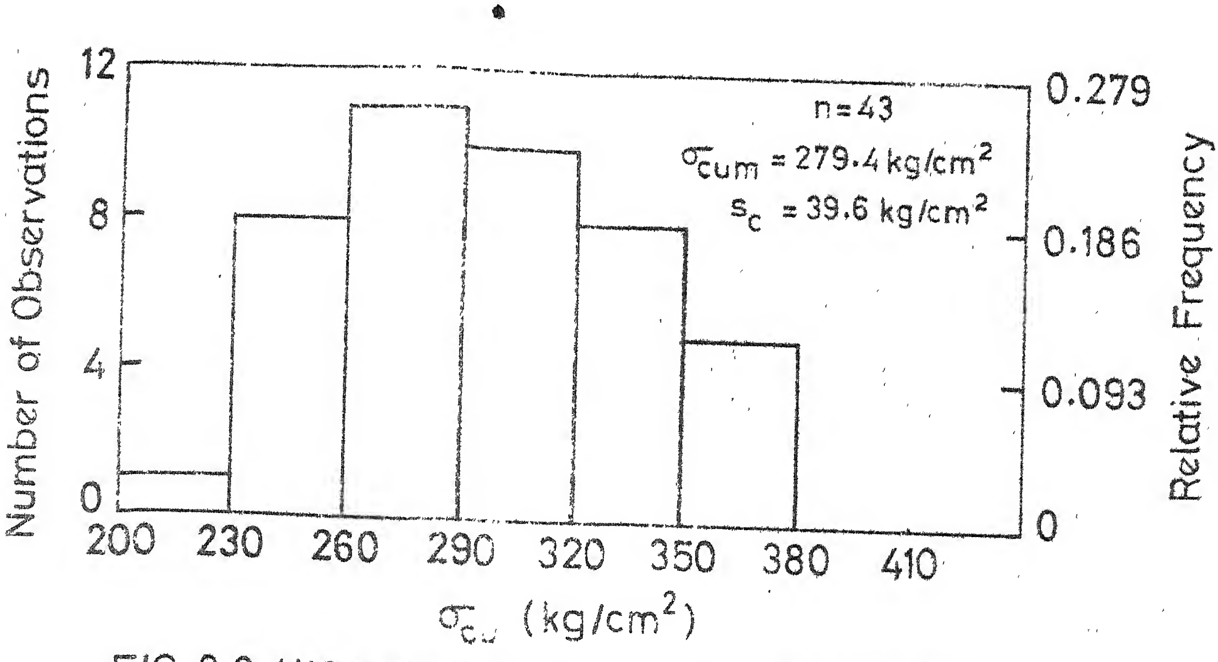


FIG. 2.6 HISTOGRAM OF M250 CONCRETE FOR A TYPICAL GROUP

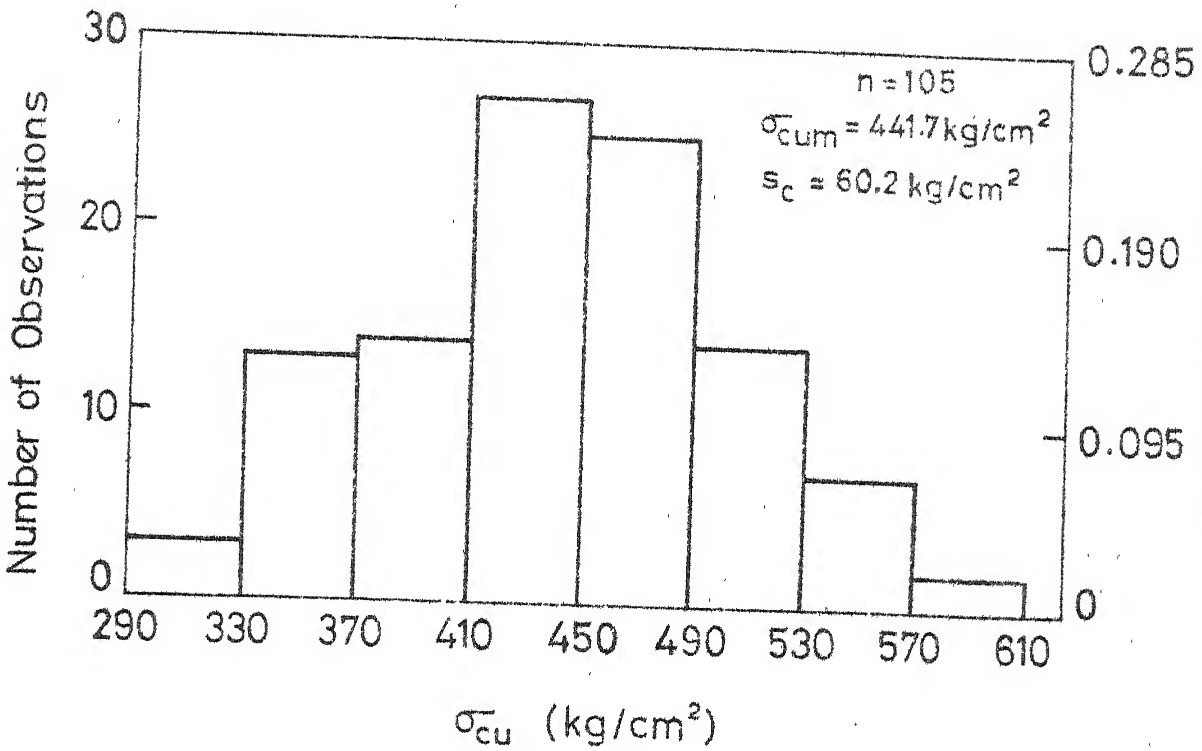


FIG.2.7 HISTOGRAM OF M350 CONCRETE FOR A TYPICAL GROUP

Figs. 2.8 to 2.11 give the histograms and comulative distributions of the strengths of the concretes. The approximate number of intervals for drawing the histograms is selected (48) as

$$z = 1 + 3.3 \log_{10} n \qquad (2.1)$$

in which z is the number of intervals and n is the number of samples. A convenient value of the interval is chosen from the appropriate value of z.

The suitability of a mathematical model to fit the data is arrived at after applying the chi square test with one percent level of significance. The value of the chi square is computed from the formula (49)

$$\chi^2 = \sum_{i=1}^{a} \frac{(X_i - Y_i)^2}{Y_i} \qquad (2.2)$$

where

χ^2 = the value of chi square

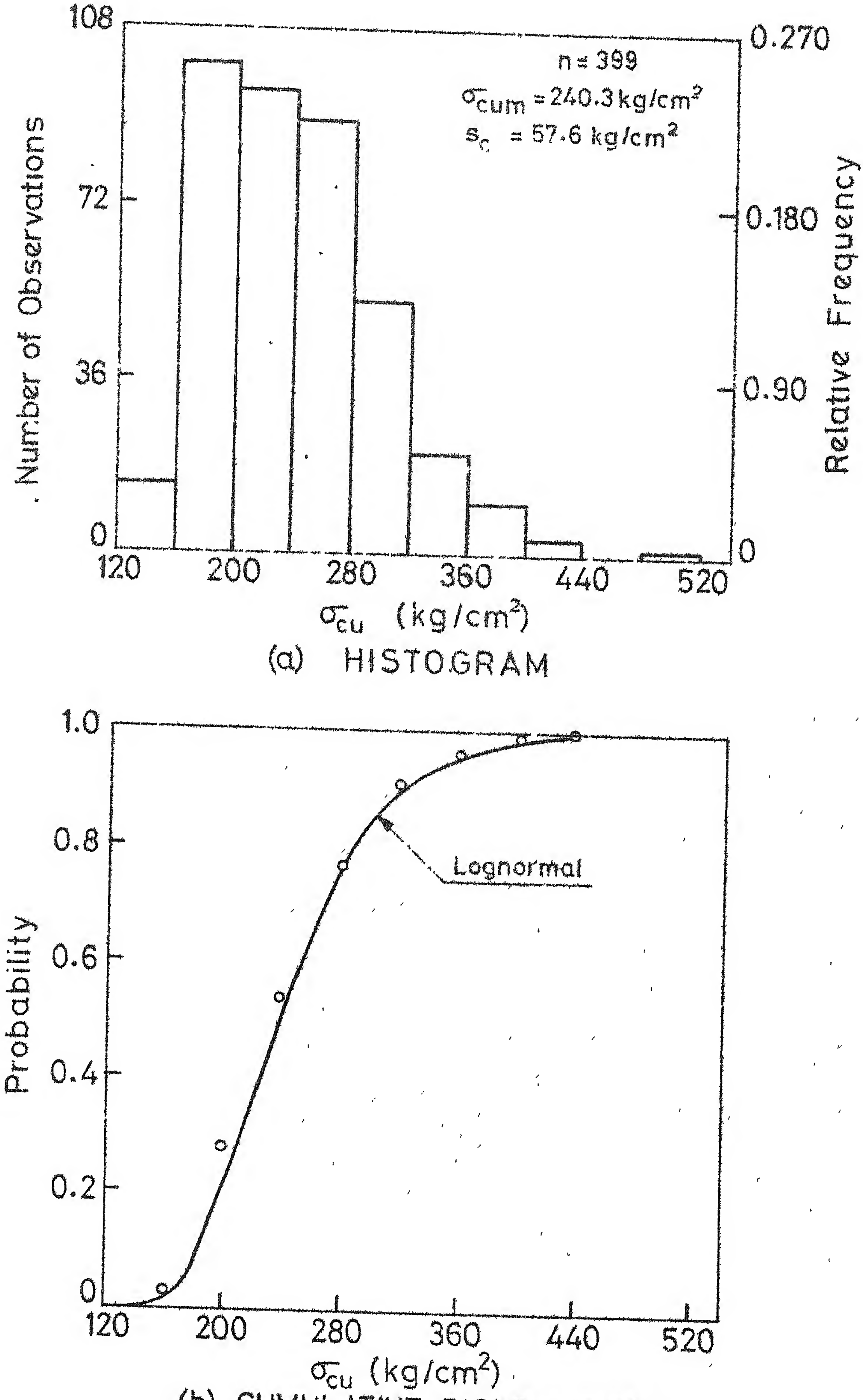
X_i = the observed frequency in the ith interval

Y_i = the expected frequency corresponding to the model in the ith interval

a = the number of categories considered.

The number of degrees of freedom is given by

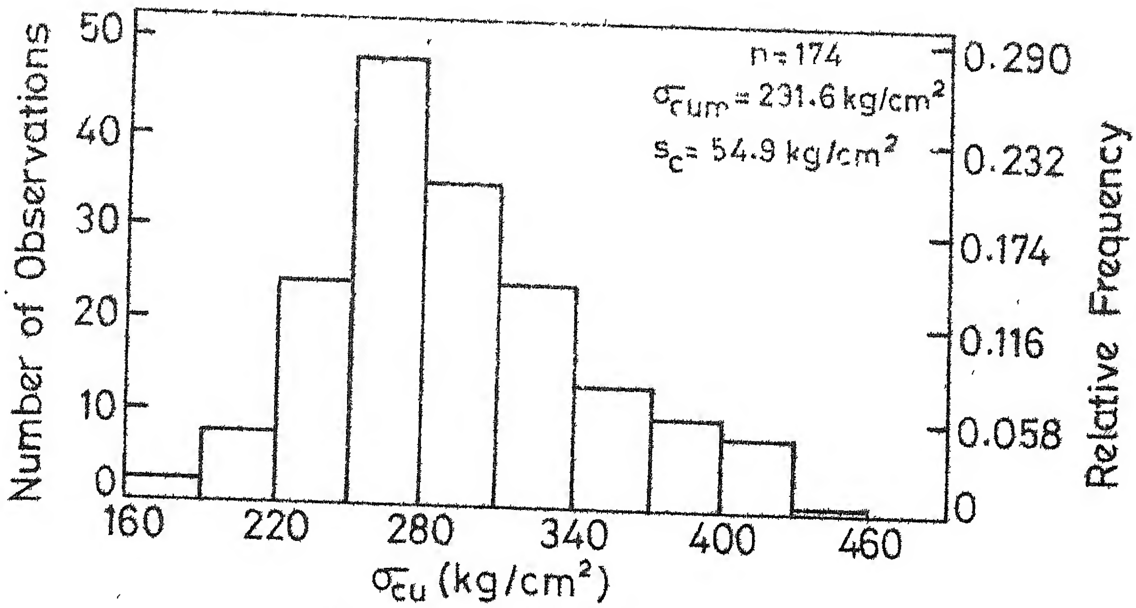
$$M = a - r - 1 \qquad (2.3)$$



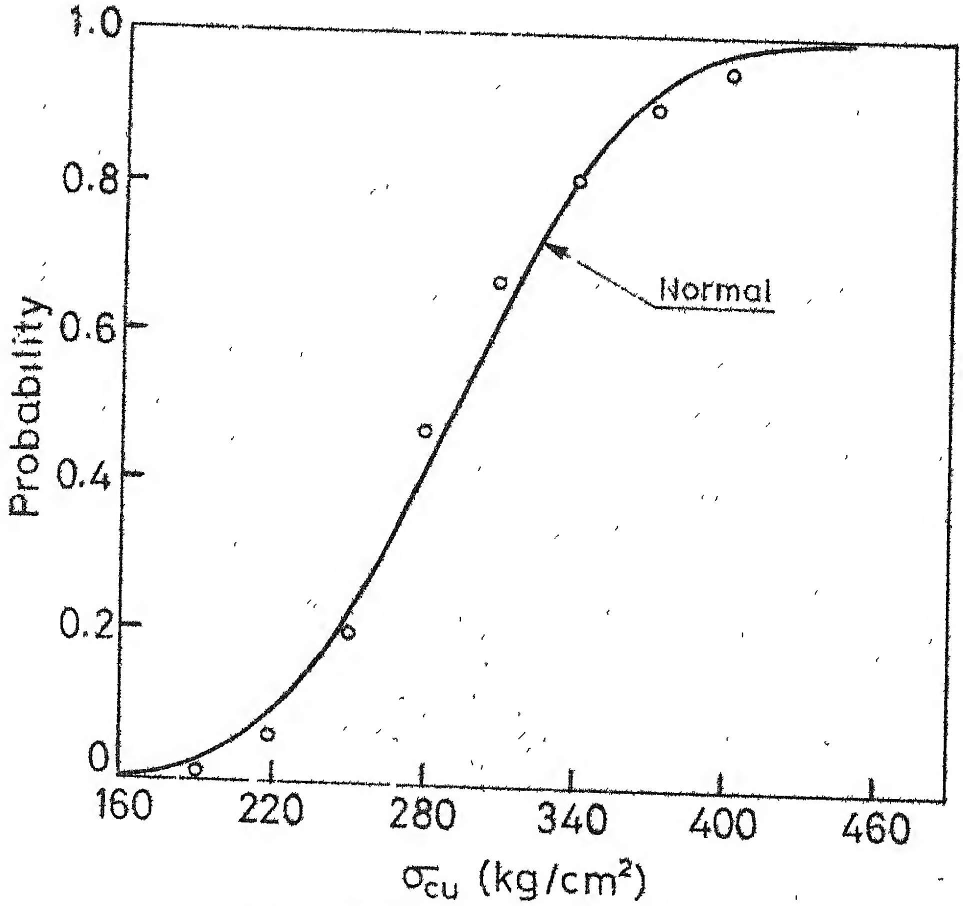
(a) HISTOGRAM

(b) CUMULATIVE DISTRIBUTION

FIG.2.8 TYPICAL CLASS OF M150 CONCRETE

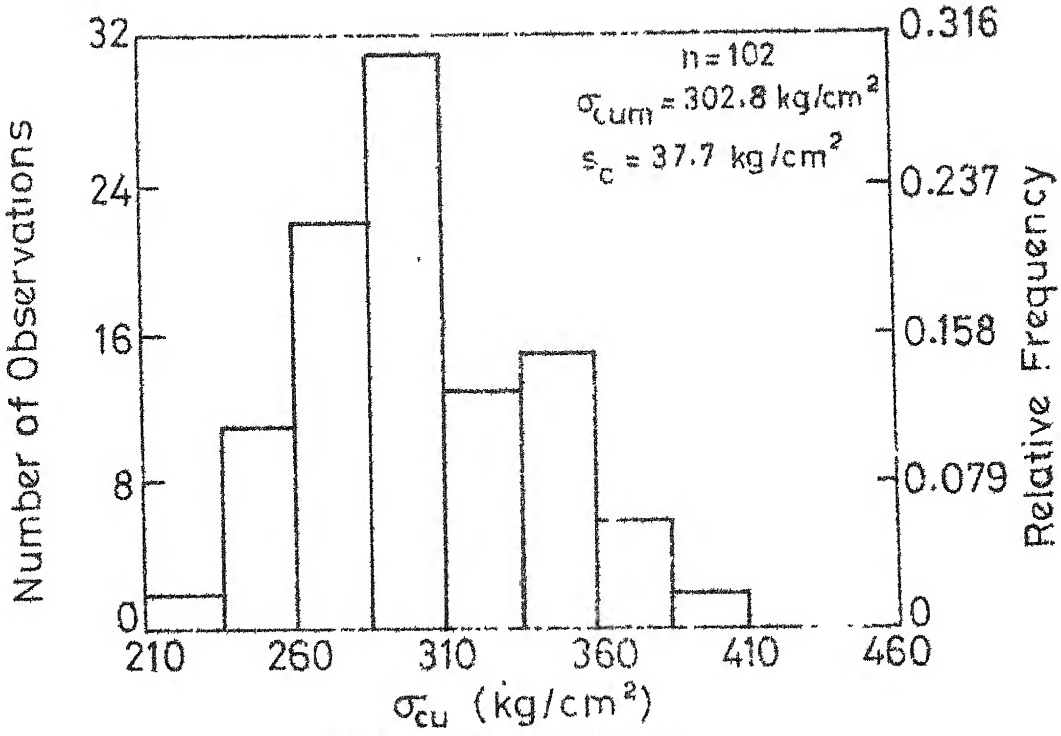


(a) HISTOGRAM

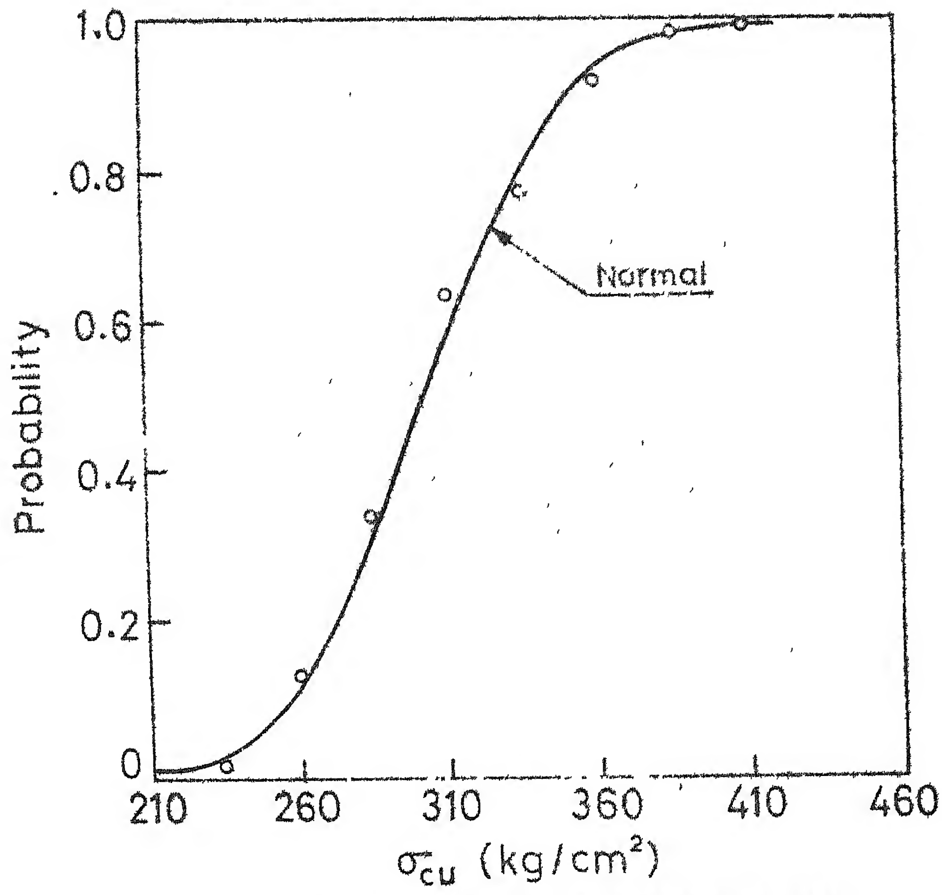


(b) CUMULATIVE DISTRIBUTION

FIG.2.9 TYPICAL CLASS OF M200 CONCRETE

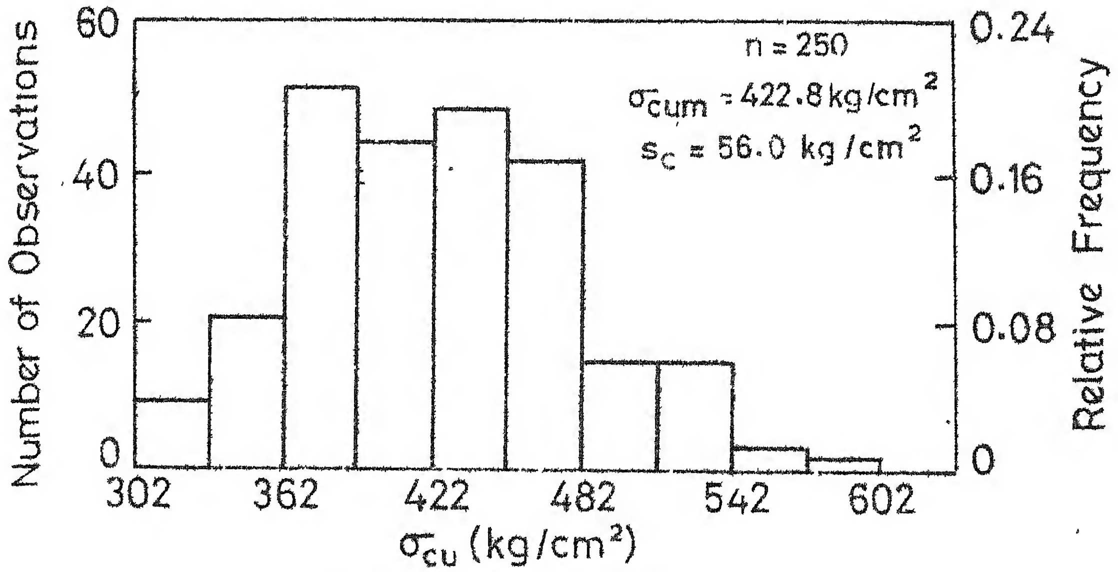


(a) HISTOGRAM

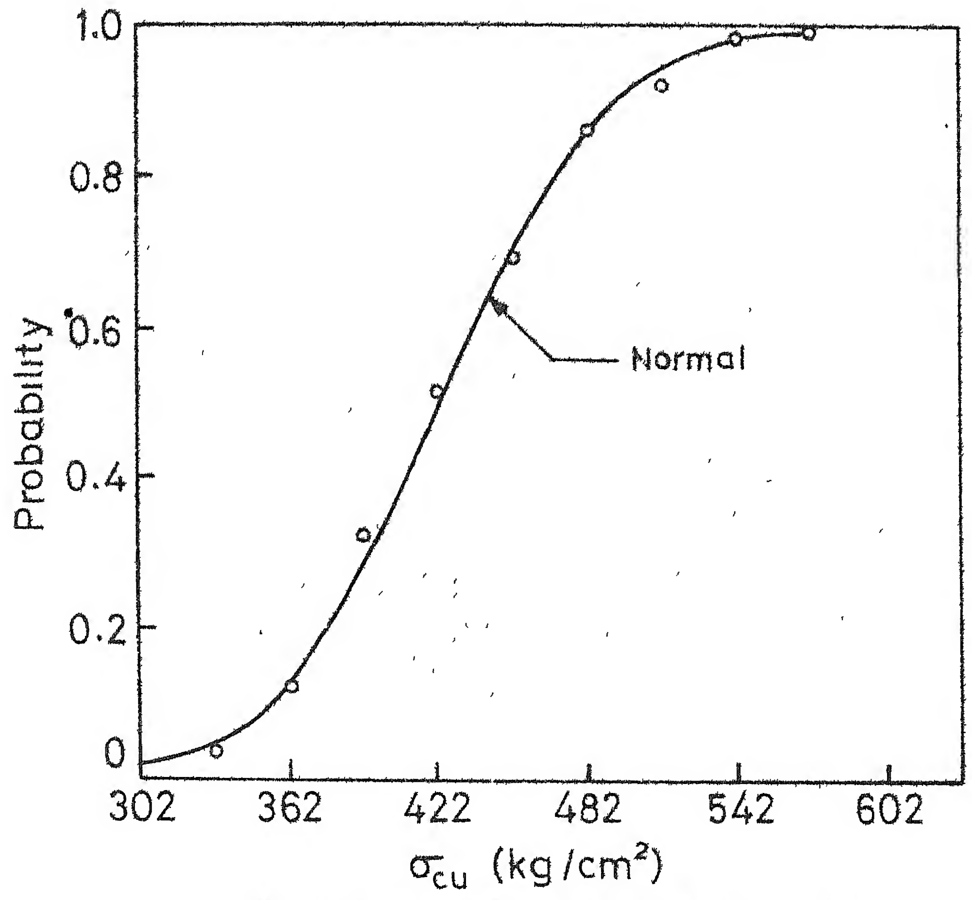


(b) CUMULATIVE DISTRIBUTION

FIG.2.10 TYPICAL CLASS OF M250 CONCRETE



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG.2.11 TYPICAL CLASS OF M350 CONCRETE

where M = the number of degrees of freedom

r = the number of parameters estimated from the data

The chi square values computed from Eq. 2.2 for the assumed distributions are checked with the standard chi square values given in the standard tables (49). The normal distribution is found to satisfy the chi square test for M200, M250 and M350 concretes at one percent level of significance. However, the normal distribution is found unsatisfactory for the complete class of M150 concrete samples, even though three out of the six groups satisfied the test. The lognormal distribution is found satisfactory for M150 concrete samples. The computed values of chi square and the values of chi square obtained from the standard tables (49) for four classes of concretes are given in table 2.1.

2.2.3 Analysis of Probability of Failure

The probability of failure of concrete is given by

$$p_{fc} = P(\sigma_{cu} < \sigma_{cuk}) = \int_{-\infty}^{\sigma_{cuk}} f(\sigma_{cu})\, d\sigma_{cu} \qquad (2.4)$$

where p_{cu} = the probability of failure of concrete cube

σ_{cu} = the strength of concrete cube at 28 days

σ_{cuk} = the characteristic strength of concrete cube

$f(\sigma_{cu})$ = the density function of strength of concrete.

For a normal density function, the Eq. 2.4 reduces to

Table 2.1. Values of chi square for four classes of concretes for one percent significance level*

Class	Computed Value	Value Obtained from the Tables	Remark
M150	2.04	15.09	Lognormal
M200	9.75	11.34	Normal
M250	7.13	15.09	Normal
M300	11.06	16.81	Normal

*M150, M250 and M350 classes of concretes also satisfy corresponding distributions for five percent significance level.

Table 2.2. Probability of failure of concrete cubes

Sl. No.	Charac-teristic Strength (kg/cm^2)	Mean Strength (kg/cm^2)	Standard Deviation (kg/cm^2)	Coefficient of Variation	Probability of Failure	Percentage of Sets Satisfying IS Code(51)	Type of Distribution
1	150	240.37	57.65	0.239	0.0332	97.08	Lognormal
2	200	291.60	54.90	0.188	0.0476	95.08	Normal
3	250	302.80	37.70	0.124	0.0808	80.00	Normal
4	350	422.80	56.00	0.131	0.0983	91.44	Normal

$$p_{cu} = \phi\left(\frac{\sigma_{cuk} - \sigma_{cum}}{s_c}\right) \qquad (2.5)$$

in which

$\phi =$ the cumulative distribution function of the standardized normal variable (50)

$\sigma_{cum}, s_c =$ parameters of the concrete namely mean value and standard deviation

σ_{cum} and s_c are expressed as (49)

$$\sigma_{cum} = \sum_{i=1}^{n} \sigma_{cui}/n \qquad (2.6)$$

$$s_c^2 = \sum_{i=1}^{n} (\sigma_{cui} - \sigma_{cum})^2/(n-1) \qquad (2.7)$$

For lognormal distribution (48), the Eq. 2.4 simplifies to

$$p_{cu} = \phi\left[\frac{\log(\sigma_{cuk}/\sigma_{cul})}{s_{cl}}\right] \qquad (2.8)$$

in which σ_{cul} and s_{cl} are the parameters of the lognormal distribution. They are expressed as

$$\sigma_{cul} = \sigma_{cum} \exp(-0.5\, s_{cl}^2) \qquad (2.9)$$

$$s_{cl}^2 = \log(\delta_c^2 + 1) \qquad (2.10)$$

where δ_c is the coefficient of variation of σ_{cu}. Table 2.2 shows the probability of failure of the concrete for different classes of concrete, computed from Eqs. 2.5 and 2.8. The percentage of the tests of concrete cubes satisfying the Indian Code specifications on quality control (51) is also listed in the same table.

2.2.4 Relation Between Characteristic and Mean Strengths of Concrete

The aim of a builder is to produce a concrete in the field whose strength is as close as possible to that of characteristic strength. However, there will be certain variation of the mean strength of the test specimens from the characteristic. One of the aims of this thesis is to find the relation between characteristic and mean value of the strength of concrete. This thesis establishes the expected probability of failure of the concrete for a given characteristic strength. Assuming the normal distribution, Eq. 2.5 gives the probability of failure of the concrete. This equation can be rearranged as

$$\left(\frac{\sigma_{cuk} - \sigma_{cum}}{s_c}\right) = \phi^{-1}(p_{cu}) = -k \tag{2.11}$$

in which k is the coefficient depending on the probability of failure p_{cu}. The Eq. 2.11 can be rewritten as

$$\sigma_{cuk} = \sigma_{cum} - k\, s_c = \sigma_{cum}(1 - k\delta) \tag{2.12}$$

or

$$\beta = 1 - k\delta \tag{2.13}$$

where

δ = the coefficient of variation

β = $\sigma_{cuk}/\sigma_{cum}$

The Eq. 2.12 yields the relationship between the mean strength

and characteristic strength for the preassigned reliability. Using normal distribution and substituting the corresponding values of σ_{cuk} and σ_{cum} of each group of samples in Eq. 2.12, the values of β and k have been computed and given in table 2.3.

It can be seen from table 2.3 that the values of k vary from 1.21 to 2.01. Therefore, it is desirable to establish a particular value of k which can be used in the design. This value of k is obtained by using the method of least squares, based on the field data. The CEB-FIP Committee has recommended a five percent probability of failure based on which the value of k works out to be 1.64 assuming normal distribution for the strength of the concrete.

2.2.5 Least Square Fit

For each group of samples, the values of β and δ are computed and shown in Fig. 2.12. The value of β is considered as a random variable for an assigned value of δ. A linear regression model connecting β and δ is selected based on the assumptions that the values of β corresponding to a given value of δ follow normal distribution and the variance of the values of β for a given δ is independent of the magnitude of δ. The equation of the model,

$$\beta = b_1 - k\delta \tag{2.14}$$

in which b_1 and k are parameters of the model, is selected in conformity with that of the equation recommended by CEB-FIP(45).

Table 2.3. Values of β and k for various groups of concrete

Group No.	β	k	δ
1	0.571	1.7463	0.2450
2	0.628	1.4386	0.2580
3	0.635	2.0100	0.1820
4	0.740	1.5173	0.1720
5	0.674	1.4289	0.2290
6	0.520	1.7561	0.2740
7	0.687	1.6650	0.1880
8	0.842	1.1967	0.1331
9	0.817	1.5704	0.1177
10	0.783	1.5243	0.1360
11	0.860	1.2100	0.1131

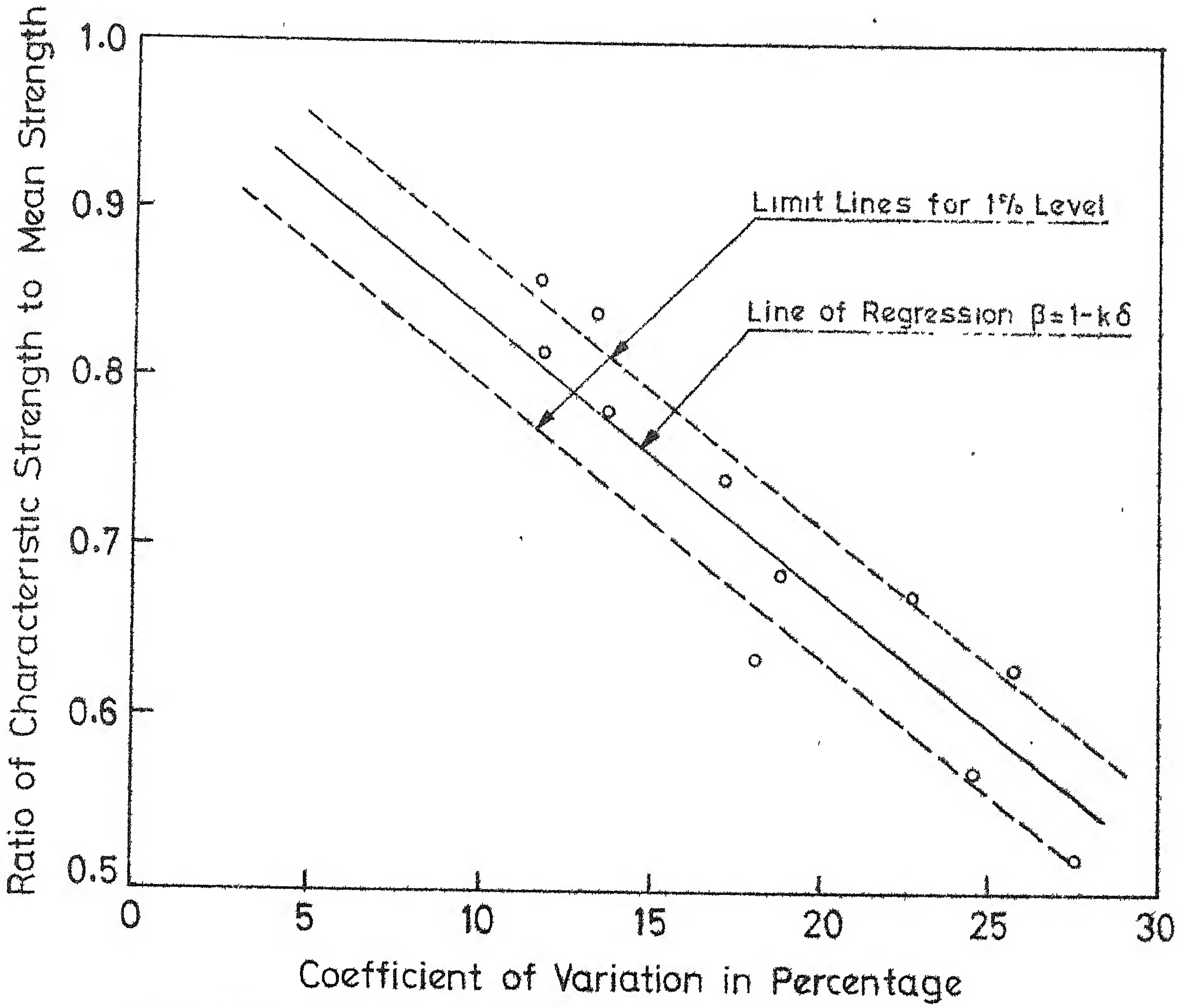


FIG. 2.12 RELATION BETWEEN δ and β

However when the variance is zero, it is known that the mean value of the test results should coincide with the characteristic value. This leads to $\beta = 1$, when $\delta = 0$. Substituting the above values of β and δ in Eq. 2.14, one gets $b_1 = 1$. Hence Eq. 2.14 reduces to the Eq. 2.13 with one parameter k. Using the method of least squares, k is chosen to minimize the sum of the squared differences between the observed values β_i and the estimated expected values given by Eq. 2.13, that is k is chosen to minimize

$$\sum_{i=1}^{N} [\beta_i - (1 - k\delta_i)]^2 \tag{2.15}$$

where N is the number of the groups of the data. Taking derivative of Eq. 2.15 with respect to k and equating to zero, the value of k is given by the following equation.

$$k = \frac{\sum_{i=1}^{N} \delta_i - \sum_{i=1}^{N} \delta_i \beta_i}{\sum_{i=1}^{N} \delta_i^2} \tag{2.16}$$

The computed value of k from Eq. 2.16 is equal to 1.6. Rearrangement of the Eq. 2.11 gives

$$p_{cu} = \phi(-k) \tag{2.17}$$

The value of p_{cu} is equal to 5.5 percent for the established value of k equal to 1.6. The line of regression of β on δ for the Eq. 2.13 is shown in the Fig. 2.12.

2.2.6 Regression Analysis

The regression analysis is done to indicate the level of confidence and the acceptability of the most probable value of the failure of concrete cubes established earlier.

The standard deviation of β, s_β, is computed from the following equation (49)

$$s_\beta^2 = \frac{1}{N-1} \sum_{i=1}^{N} [\beta_i - (1-k\delta_i)]^2 \tag{2.18}$$

and it is found to be 0.0412. The variance of the mean value of β, $s_{\beta m}^2$, is given by

$$s_{\beta m}^2 = \frac{s_\beta^2}{N} \tag{2.19}$$

The confidence interval for the estimate of the mean value of β is given by

$$\beta_m - t\, s_{\beta m} \leq \bar{\beta}_m \leq \beta_m + t s_{\beta m} \tag{2.20}$$

in which

β_m = the mean value of β

t = the 't' statistic

$\bar{\beta}_m$ = the possible range of β_m.

The number of degrees of freedom of the problem is equal to (N-1). For one percent level of significance, the confidence interval for the estimate of β_m is found from the standard table (49) and it

varies from 0.6659 to 0.7499. The corresponding confidence limit lines are shown in Fig. 2.12.

The variance of k is computed using the value of s_β in the following equation

$$s_k^2 = \frac{s_\beta^2}{\sum_{i=1}^{N} (\delta_i - \delta_m)^2} \tag{2.21}$$

in which s_k^2 = the variance of k

δ_m = the mean value of δ .

The confidence interval for the estimate of k is given by

$$k - ts_k \leq \bar{k} \leq k + ts_k \tag{2.22}$$

in which $\bar{k}$ is the possible range of k. For one percent level of significance and ten degrees of freedom, the confidence interval for the estimate of k is from 0.880 to 2.320. It can be seen from the table 2.3 that all the values of k for different groups of the data lie within the upper and lower confidence limits. The result of the significance test establishes the credibility of the estimated value of k.

The correlation coefficient ρ is calculated from the formula (49)

$$\rho = \frac{\sum_{i=1}^{N} \delta_i \beta_i}{\sqrt{\sum_{i=1}^{N} \delta_i^2 \sum_{i=1}^{N} \beta_i^2}} \tag{2.23}$$

and its value is equal to 0.91. For ten degrees of freedom the value of ρ , obtained from standard tables (49) for one percent level of significance, is less than the calculated value. Hence there is correlation between β and δ .

2.3 STATISTICAL ANALYSIS OF STRENGTH OF STEEL

2.3.1 Histogram and Statistical Analysis

The structural engineering laboratory at IIT Kanpur has conducted number of tests on strengths of different steels. This thesis presents statistical analysis of the data on strengths of high tensile steel (HTS) and cold twisted deformed (CTD) steel.

There are seven groups of data on 7mm ϕ HTS wire. Number of specimens in each group varies from 2 to 22. All the samples belonging to each group are combined and the histogram and cumulative distribution of the ultimate strength of 7mm ϕ HTS wire are given in Fig. 2.13. The normal distribution is found to satisfy the chi square test at one percent level of significance (satisfies for 5%. significance level also).

Forty two specimens of 5mm ϕ HTS wire belonging to one coil have been tested. The histogram and cumulative distribution of the ultimate strength of 5mm ϕ HTS wire are given in Fig. 2.14. Normal distribution is found to satisfy the chi square test at one percent level of significance (satisfies for 5% significance level also).

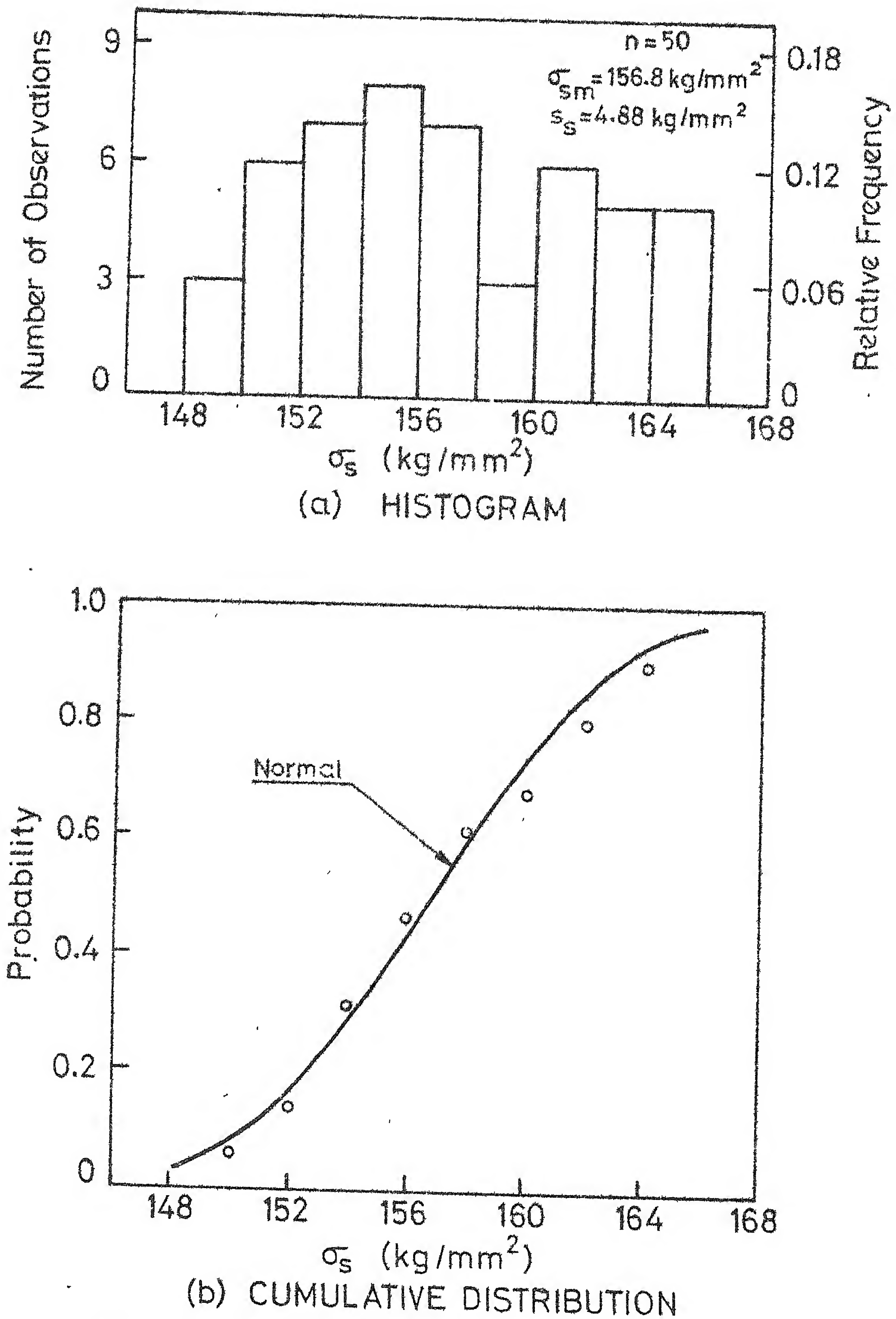
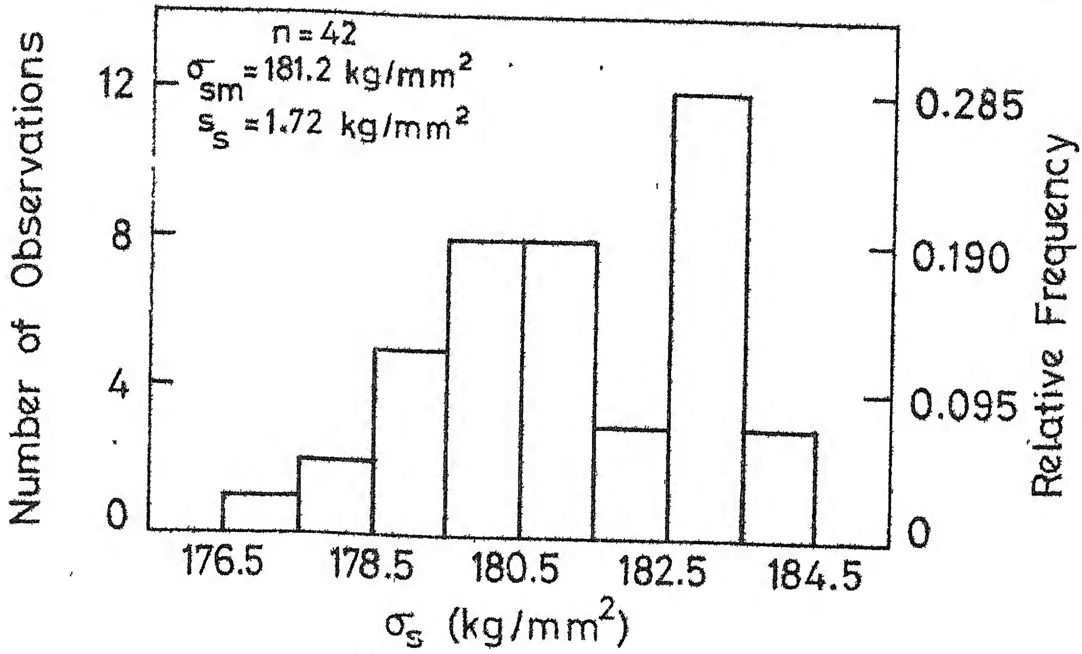
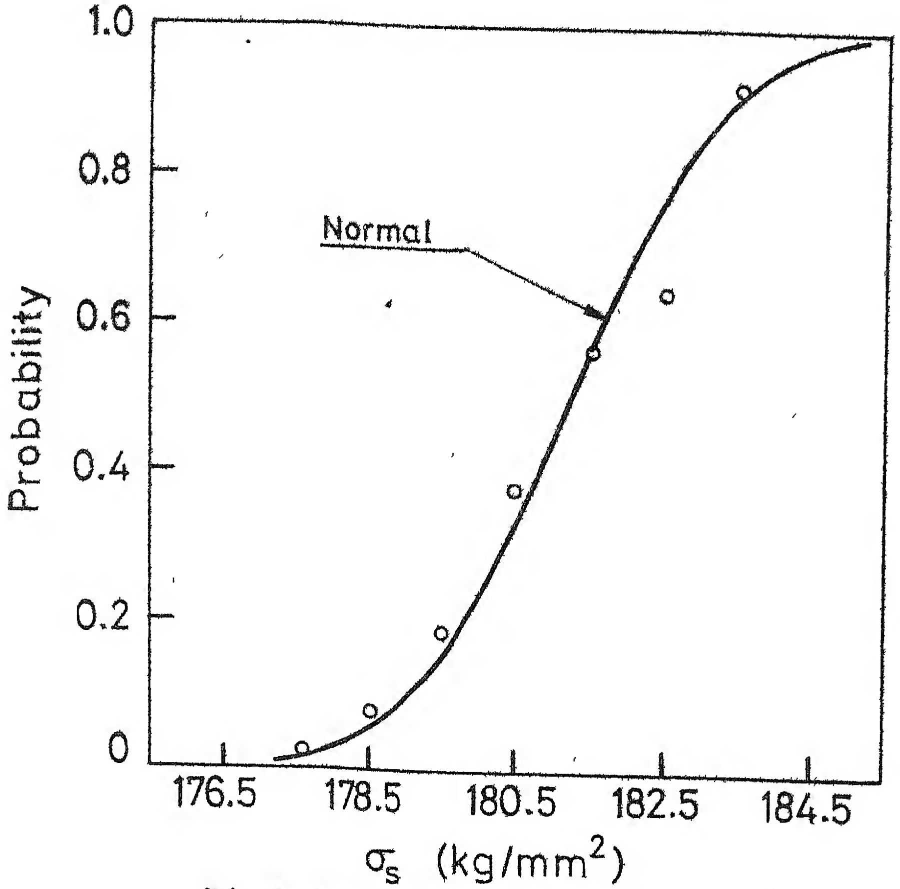


FIG. 2.13 HIGH TENSILE STEEL (7 mm ϕ)



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG. 2.14 HIGH TENSILE STEEL (5mm ϕ)

There are 8 sets of data on strength of CTD steel. Number of specimens in each set is 6. Nominal diameter of specimens is equal to 25mm for four sets, 28mm for 2 sets and 32mm for one set. As the characteristic strength of CTD steel upto 32mm ϕ is same (52), all sets have been combined to form a group. Ultimate strength and proof strength have been recorded. Histogram and cumulative distribution of the data on proof strength and ultimate strength of torsteel are given in Figs. 2.15 and 2.16 respectively. Normal distribution is found to satisfy the chi square test for the data on proof strength at one percent level of significance. It has not been possible to fit any standard distribution for the data of Fig. 2.16.

2.3.2 Analysis of Probability of Failure

For a normal density function of ultimate strength of steel, the probability of failure of steel, p_s, is given by

$$p_s = \phi\left(\frac{\sigma_{sk} - \sigma_{sm}}{s_s}\right) \tag{2.24}$$

where σ_{sk}, σ_{sm} and s_s are the characteristic value, mean value and standard deviation of the ultimate strength of steel respectively. Table 2.4 shows the probability of failure of HTS wires and CTD steel computed from Eq. 2.24. It is observed that the characteristic strength specified by I.S. Code (46) for 5mm ϕ HTS wire is very much on the conservative side. The probability of failure for proof strength of CTD steel is observed to be high.

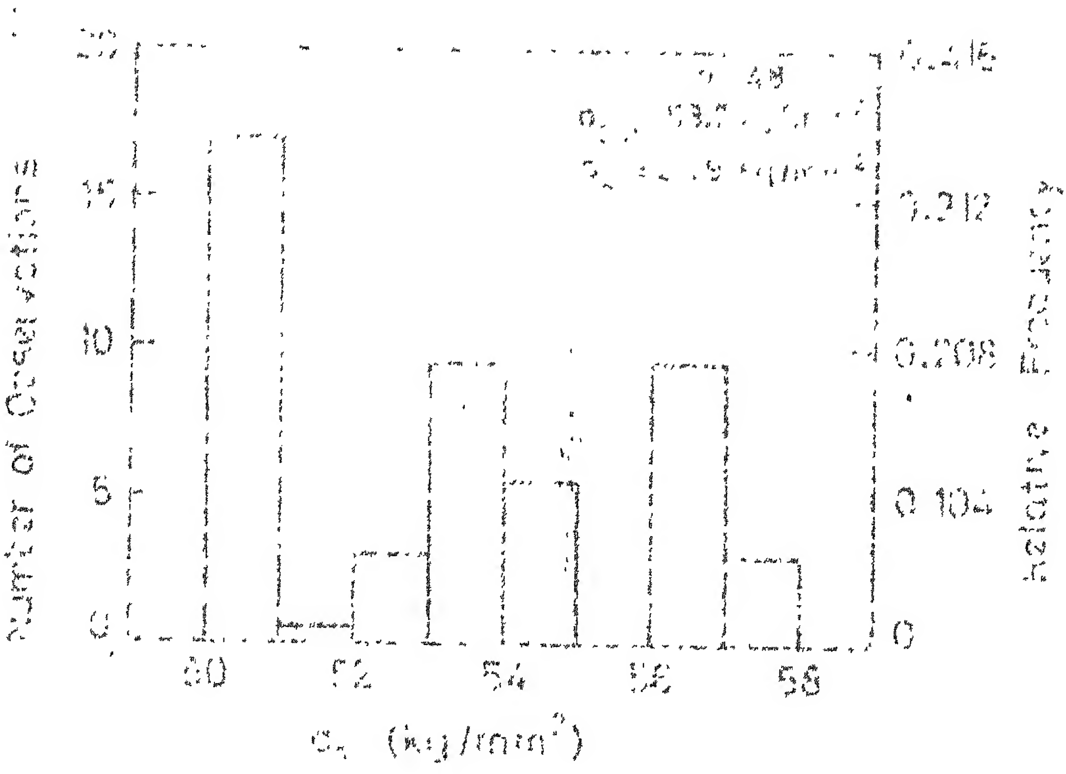


FIG. 2.16 HISTOGRAM OF σ_s OF CTD STEEL

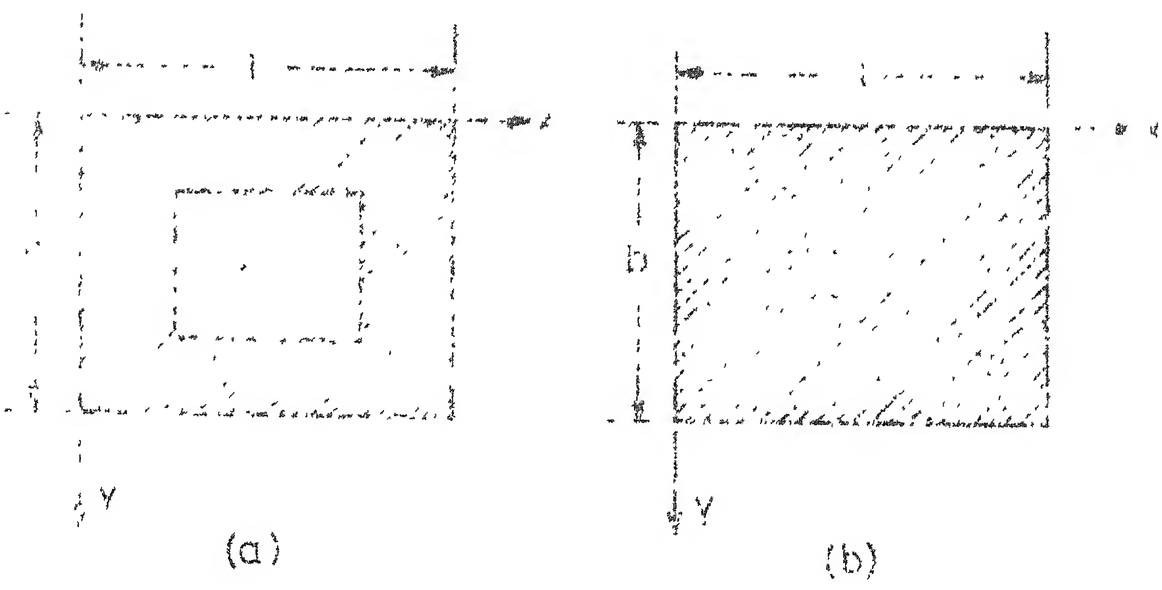


FIG. 2.17 FLOOR LOADING

Table 2.4. Probability of failure of steel

Sl. No.	Characteristic Strength (kg/mm^2)	Mean Strength (kg/mm^2)	Standard Deviation (kg/mm^2)	Coefficient of Variation	Probability of Failure	Type of Distribution	Remark
1	150	156.8	4.88	0.0311	0.0823	Normal	ultimate strength
2	160	181.2	1.72	0.0095	$< 10^{-24}$	Normal	ultimate strength
3	49.5	53.71	2.29	0.0426	-	-	ultimate strength
4	42.5	44.17	2.43	0.055	0.2451	Normal	Proof strength

2.4 STATISTICAL ANALYSIS OF LOADS

A survey of floor loads in office rooms has been carried out. The live loads in the survey include the occupants, the movable partitions, all furniture and their contents and other miscellaneous items viz typewriters, table fans etc. A volume estimate has been made of the contents of cabinets, desks and book cases and a note has been made of the nature of the contents. After observing the arrangement of files, stationeries and other items in a few almairahs, the maximum volume of an almairah that is usually used has been found to be 67 percent. All the rooms have been surveyed during peak working hours to record the number of persons working and visitors and the furniture and storage in each room. The weight of individual items have been assessed as follows. Chairs and tables have been weighed. Weight of empty steel almairah has been obtained from the manufacturer. The almairah has been assumed to be occupied by paper to an estimated volume of 67 percent of its total capacity. An average weight of a person has been estimated after knowing the weight of a few persons. Other items either have been weighed or assessed. The total load has been caloulated after adding the weight of individual items.

The magnitude of the response of floor changes with the position of the load and its distribution. Concentrated load effect should be taken into account in determining the static

equivalent floor load. An analytical study has been made for different magnitudes and occupancies of load on the floor to determine the static equivalent load. Treating floor as a plate, simply supported on all four sides as shown in Fig. 2.17, the deflection, u, of the floor at any point (x,y) can be represented by

$$u = C_1 \sin \frac{\pi x}{\ell} \sin \frac{\pi y}{b} \tag{2.25}$$

where C_1 is the coefficient of the series defining the deflection surface and ℓ and b are length and breadth of floor. During load survey it has been observed that all heavy loads are arranged near the walls of the room. Assuming 'A' area of the floor is occupied as shown in Fig. 2.17a, consider two systems of loads as shown in Figs. 2.17a and 2.17b. The hatched portions shown in Figs. 2.17a and 2.17b represent the area of the floor occupied by loads. The total load is same in both cases. By the method of virtual work concept the effect of both loads can be compared and the static equivalent load, Q_e, distributed over the whole area of the floor can be found out. The virtual work done, V, by the load Q, distributed over the area 'A' as shown in Fig. 2.17a is given by

$$V = \frac{C_1 Q}{A} \iint_A \sin \frac{\pi x}{\ell} \sin \frac{\pi y}{b}\, dx\, dy \tag{2.26}$$

The hatched area in Fig. 2.17a represents the area of integration. For fifty percent of the total area of the floor occupied by the load as shown in Fig. 2.17a, the value of V, computed from Eq. 2.26,

is $\left(\frac{1.648\ C_1 Q}{\pi^2}\right)$. The virtual work done, V_e, by Q_e (Fig. 2.17b) is

$$V_e = \frac{C_1 Q_e}{b\ell} \int_0^{\ell} \int_0^{b} \sin \frac{\pi x}{\ell} \sin \frac{\pi y}{b}\, dx\, dy$$

$$= \frac{4 C_1 Q_e}{\pi^2}$$

Equating $V = V_e$, the value of Q_r, the ratio of Q_e to Q, is obtained as 0.412. Similarly, the values of Q_r have been calculated and given in table 2.5. During load survey, as stated earlier, heavy loads are close to walls and it has been observed that load distribution in most of the rooms belong to cases 6 and 7 of table 2.5. The value of Q_r for these cases is less than 1. However, to be on the safer side, the value of Q_e is taken as equal to Q i.e. the total load is considered to be uniformly distributed over the whole area of the room (This calculation of Q_e is valid when floors are supported by walls. In the caseof large halls where partition walls are built and floors are designed as continuous slabs and flat slabs, the value of Q_r will be greater than one). Based on this, equivalent uniformly distributed floor load has been calculated by dividing the total load in each room by its area. The details of room usuage, total load, area of the room and the equivalent uniformly distributed load in each room are given in table 1 in Appendix . The areas of rooms vary from $14 m^2$ to $147.8 m^2$. The equivalent load intensity varies from $56\ kg/m^2$ to $293\ kg/m^2$. The

Table 2.5. Values of Q_r for different distribution of loads on floor

Case	Load Distributed Close to Walls		Load Distributed at the Middle Portion		Q_r
	Area	Magnitude	Area	Magnitude	
1	50%	Q	NIL	-	0.412
2	75%	Q	NIL	-	0.557
3	50%	0.75Q	50%	0.25Q	0.706
4	50%	0.50Q	50%	0.50Q	1.0
5	60%	0.50Q	40%	0.50Q	1.125
6	60%	0.75Q	40%	0.25Q	0.812
7	75%	0.90Q	25%	0.10Q	0.934

influence of room area on load is not considered. Fig. 2.18 presents histogram and cumulative distribution for equivalent uniformly distributed load in office rooms such as accounts, registrars office, departmental office etc. at IIT, Kanpur. Lognormal distribution is found to satisfy the chi square test at one percent level of significance.

Indian Standard (53) specifies a characteristic live load of 250 to 400 kg/m^2 for office buildings. If Q_k is the characteristic live load and Q_m and s_q are the parameters of the live load following lognormal distribution

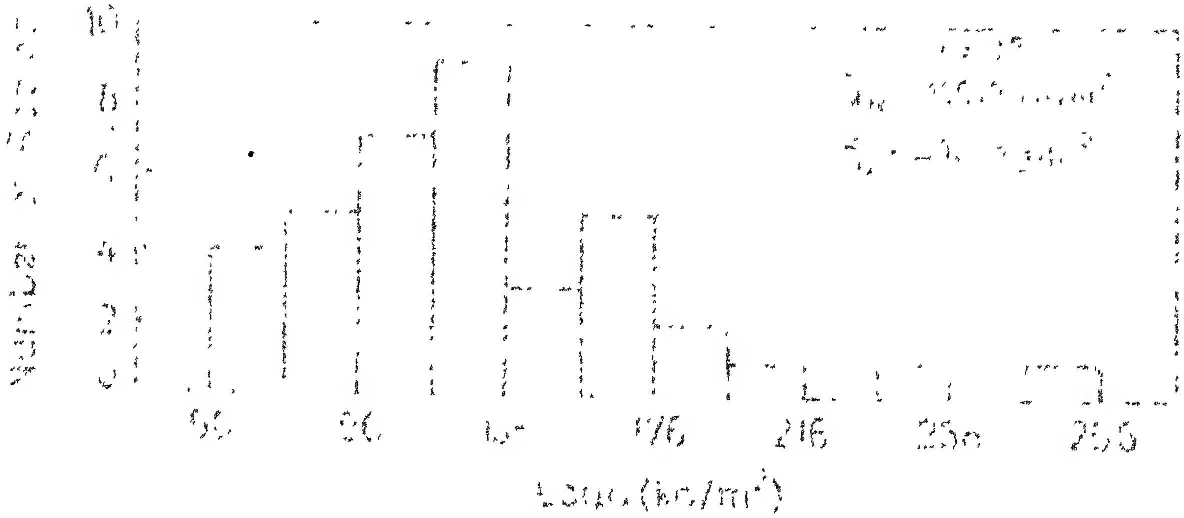
$$P(Q > Q_k) = p_q = 1 - \phi\left[\frac{\log(Q_k/Q_m)}{s_q}\right] \tag{2.27}$$

in which p_q is the probability of the load greater than Q_k. Parameters Q_k and s_q are given by (48)

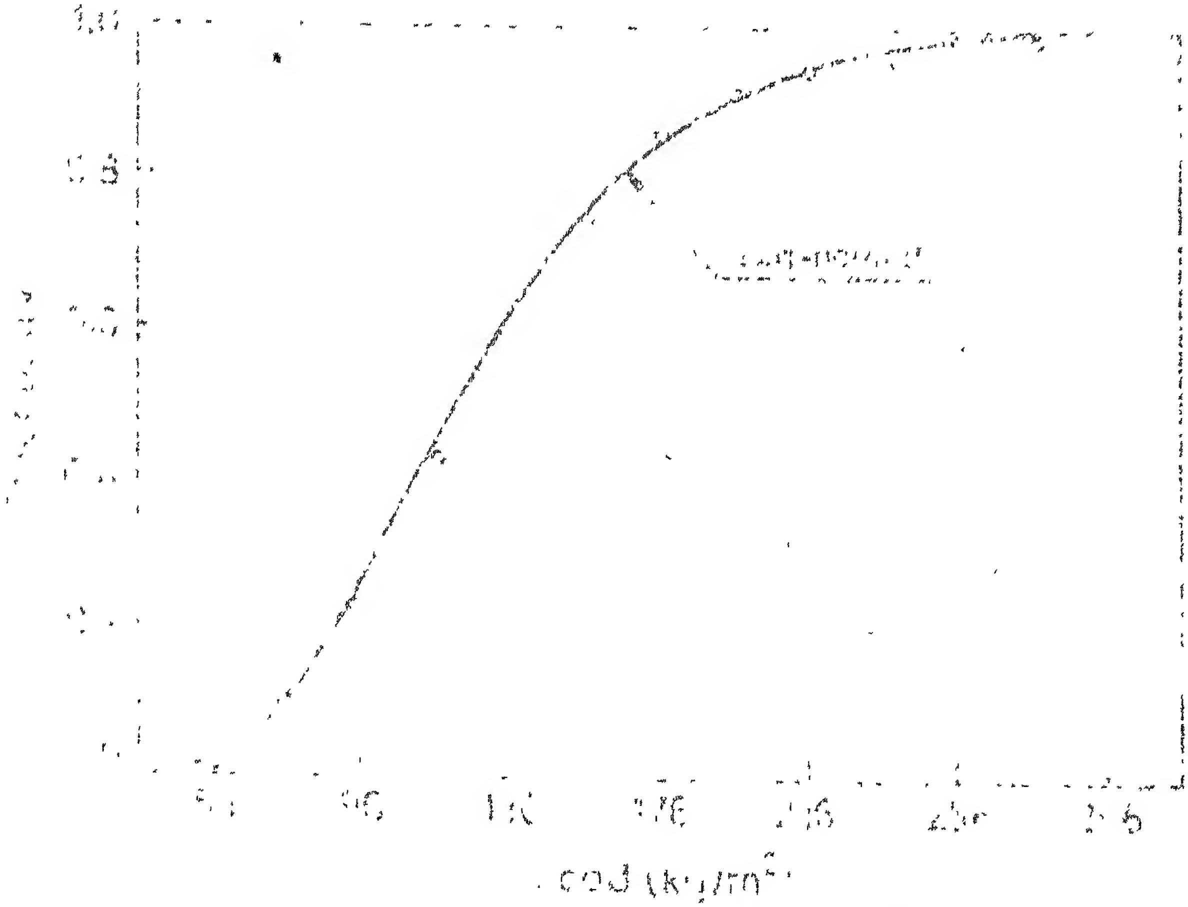
$$s_q^2 = \log \ (\delta_q^2 + 1) \tag{2.28}$$

$$Q_m = \bar{Q}_m \exp \ (-0.5 \ s_q^2) \tag{2.29}$$

where δ_q and $\bar{Q}_m$ are the coefficient of variation and mean value of the live load respectively. Their values are 0.381 and 130.3 kg/m^2. Using Eqs. 2.28 and 2.29, Q_m and s_q are calculated as 121.7 kg/m^2 and 0.368 respectively. Substituting the above values in Eq. 2.27 and assuming Q_k is equal to 400 kg/m^2, the value of p_q is given by



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG. [illegible] EQUIVALENT UDL IN OFFICE ROOMS AT I.I.T. KANPUR

$$p_q = 1 - \phi\left[\frac{\log(400/121.7)}{0.368}\right] = 6 \times 10^{-4}$$

2.5 STATISTICAL ANALYSIS OF GEOMETRIC PROPERTIES OF A SECTION

A number of prestressed concrete beams of the section shown in Fig. 2.19 have been cast and tested in the structural engineering laboratory at IIT, Kanpur for a research project. Measurements have been made to the accuracy of 0.02 mm on the geometric properties viz. width of flange at top (b_t), thickness of flange (t), thickness of web (b'), total depth (h), depths of prestressing steels (d_1, d_2, d_3) of the section. The histogram and cumulative distribution of the data of b_t, b', t_1, d_1 and h are given in Figs. 2.20 to 2.24. The normal distribution has been found to satisfy the chi square test at one percent level of significance in all the cases. It has been found that the coefficient of variation of b_t, b' and h are negligible and significant variation has been observed only in the thickness of flange and the depth of cable d_1. Specified value, mean value, standard deviation, coefficient of variation and the ratio of the mean value to its specified value of all parameters of the section are given in table 2.6. For the purpose of reliability analysis and design of beams, the variation of thickness of flange at top, t_t, is taken as the average variation of t_1 and t_2 and the variation of effective depth, d, as the average variation of d_1, d_2 and d_3.

For considering the probabilistic variation of area of steel in the reliability analysis and design of prestressed concrete beams,

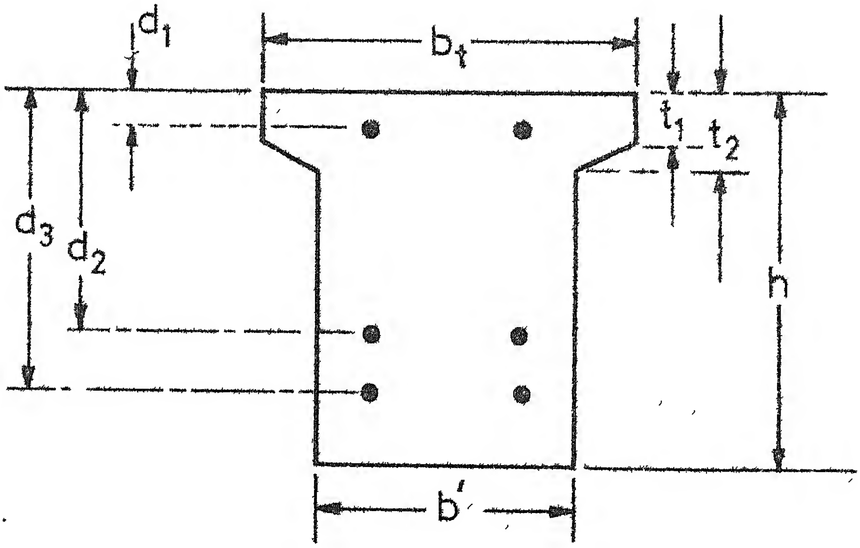
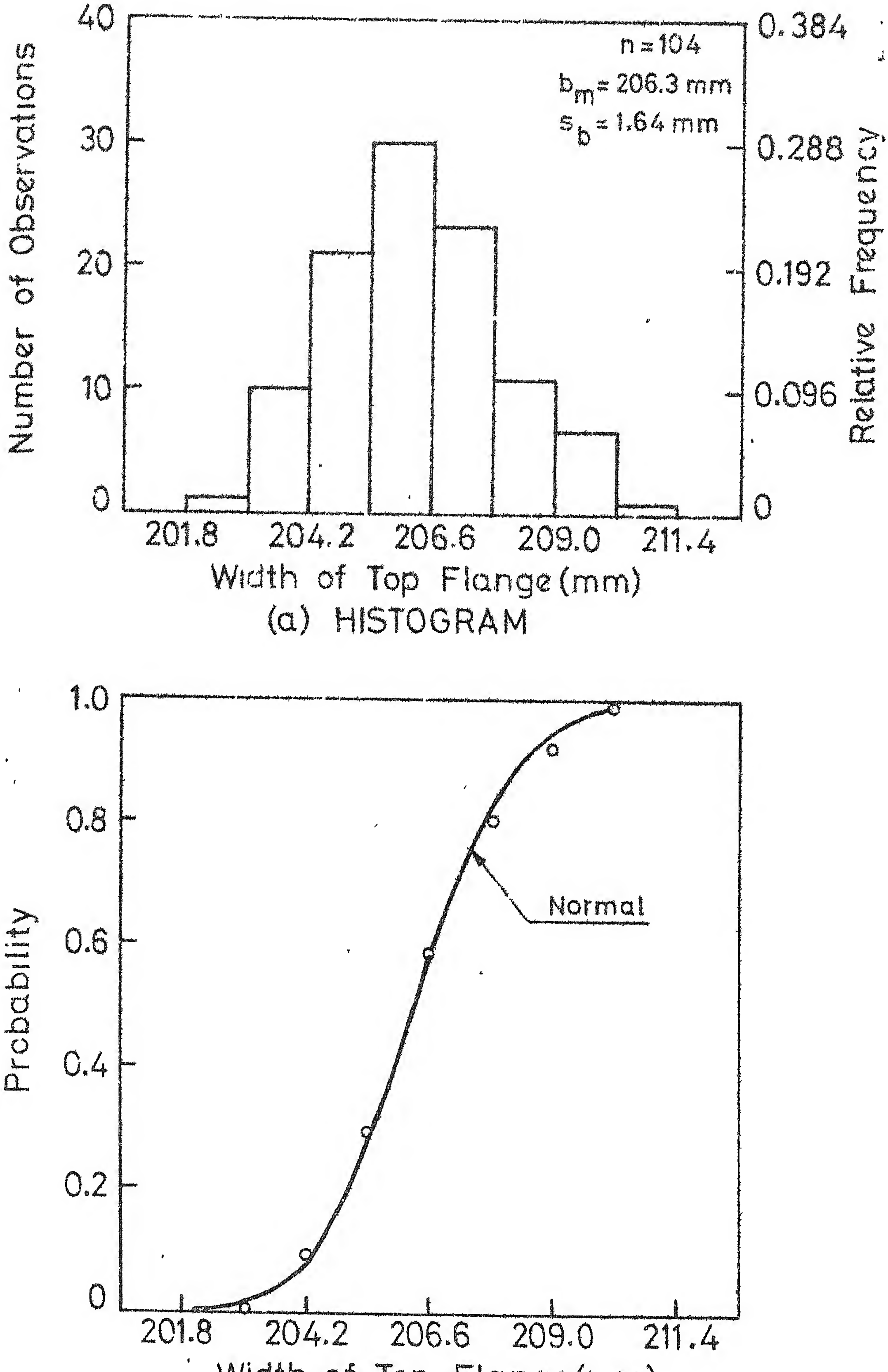


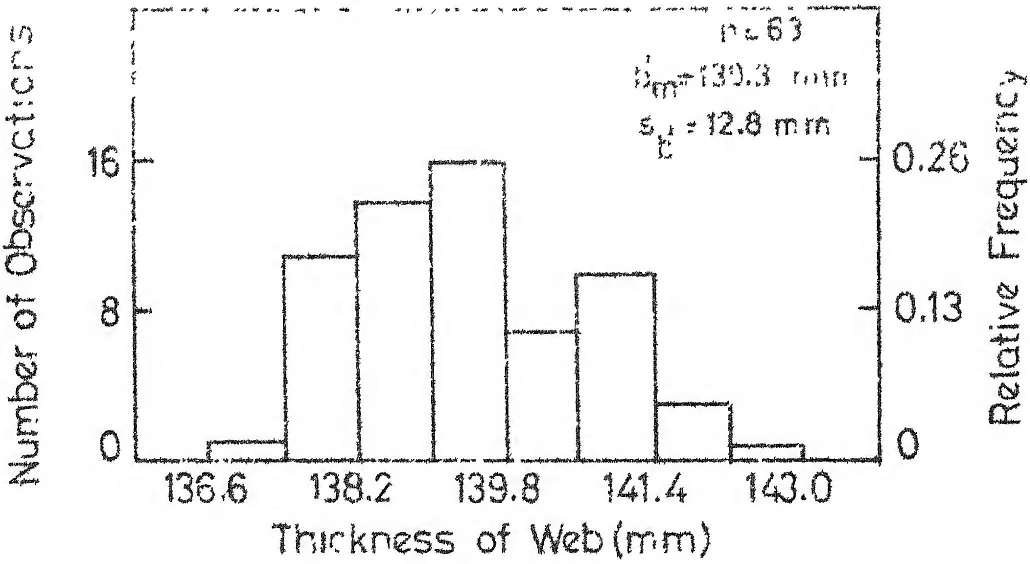
FIG. 2.19 TYPICAL SECTION



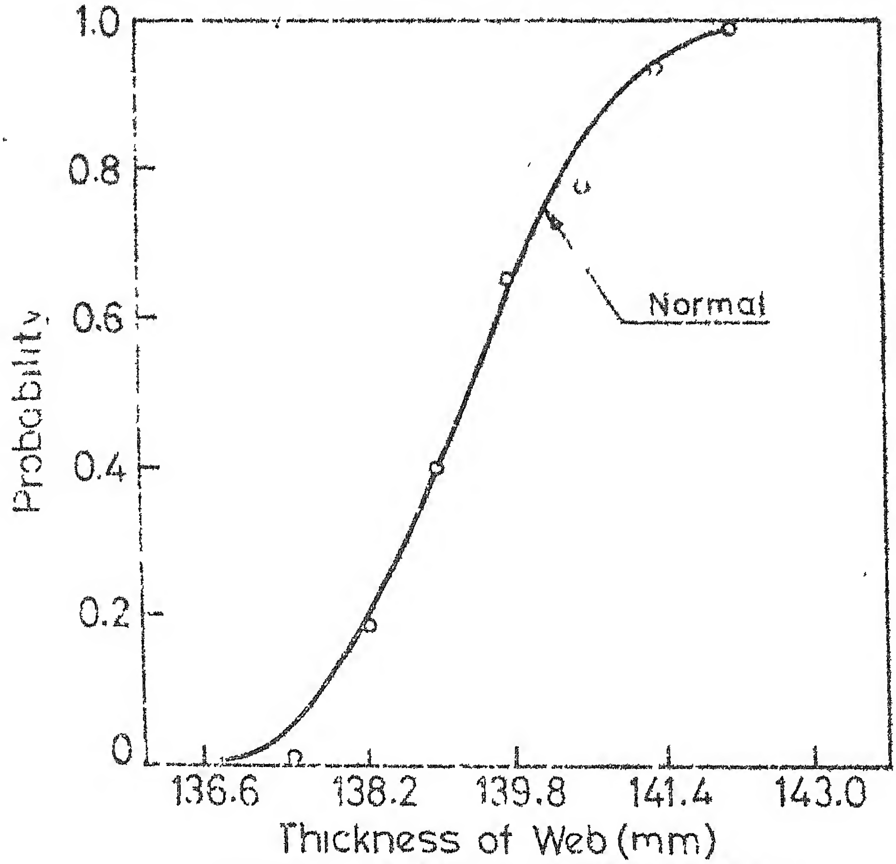
(a) HISTOGRAM

(b) CUMULATIVE DISTRIBUTION

FIG. 2.20 VARIATION OF WIDTH OF TOP FLANGE OF THE SECTION SHOWN IN FIG. 2.19

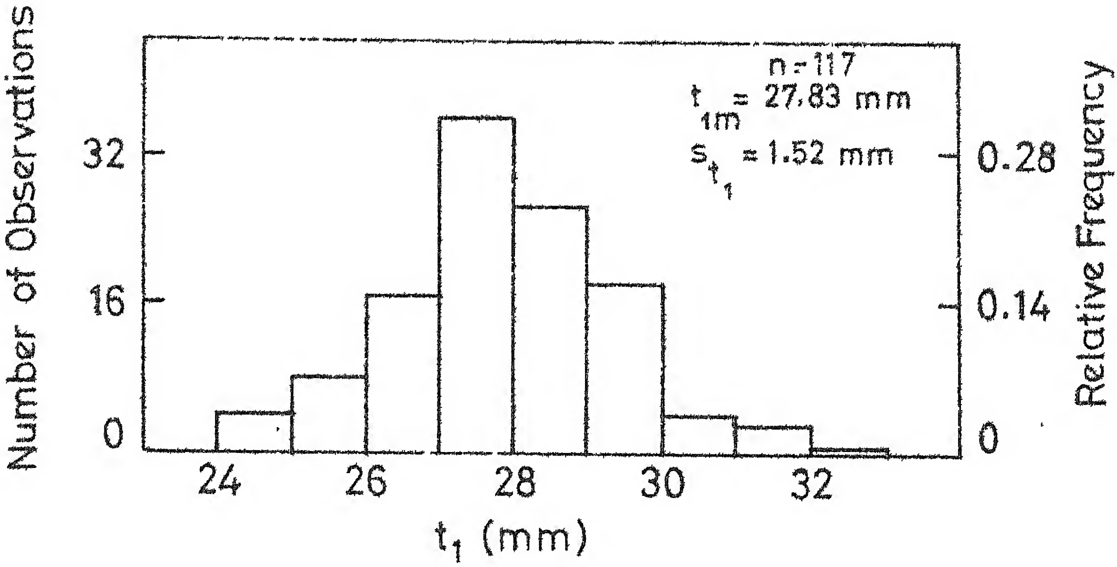


(a) HISTOGRAM

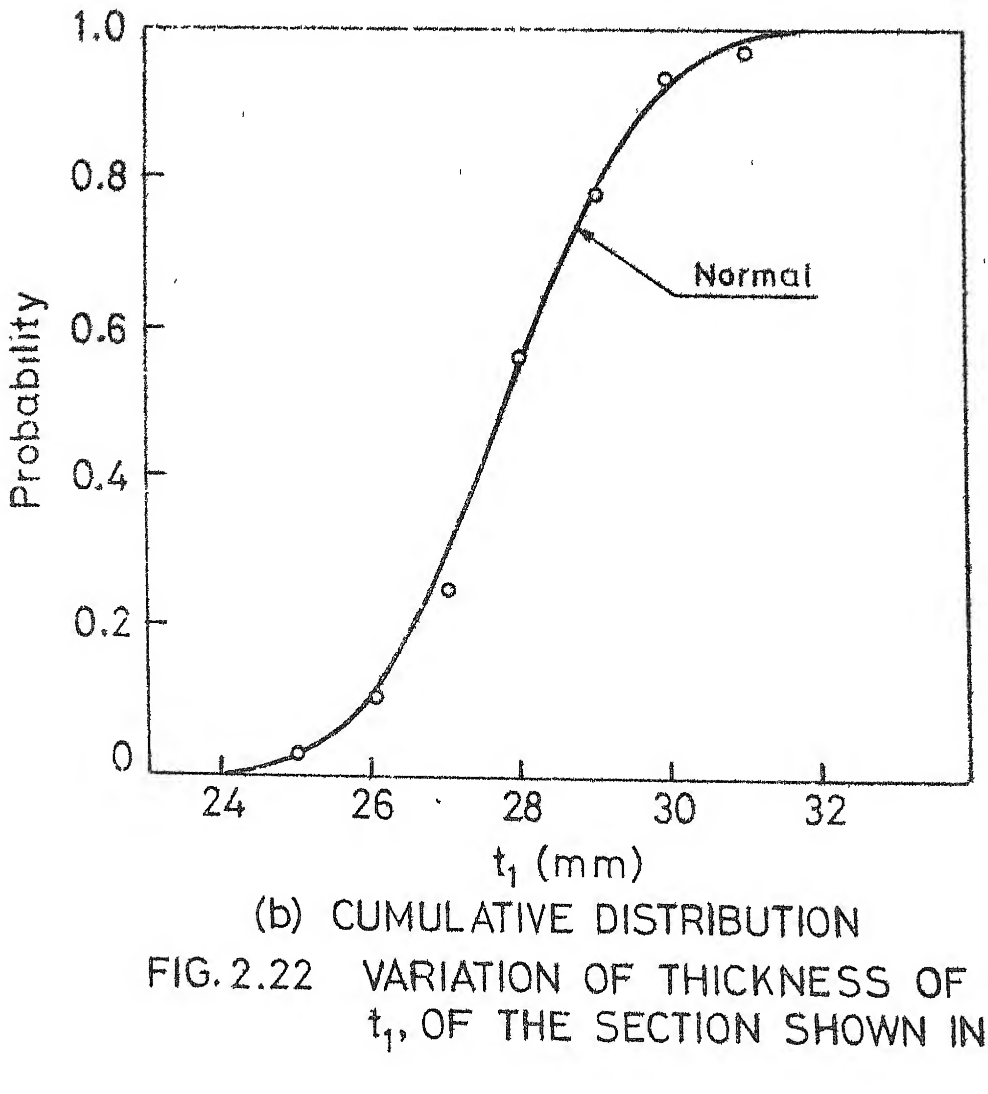


(b) CUMULATIVE DISTRIBUTION

FIG. 2.21 VARIATION OF THICKNESS OF WEB OF THE SECTION SHOWN IN FIG. 2.19



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG. 2.22 VARIATION OF THICKNESS OF FLANGE, t_1, OF THE SECTION SHOWN IN FIG. 2.19

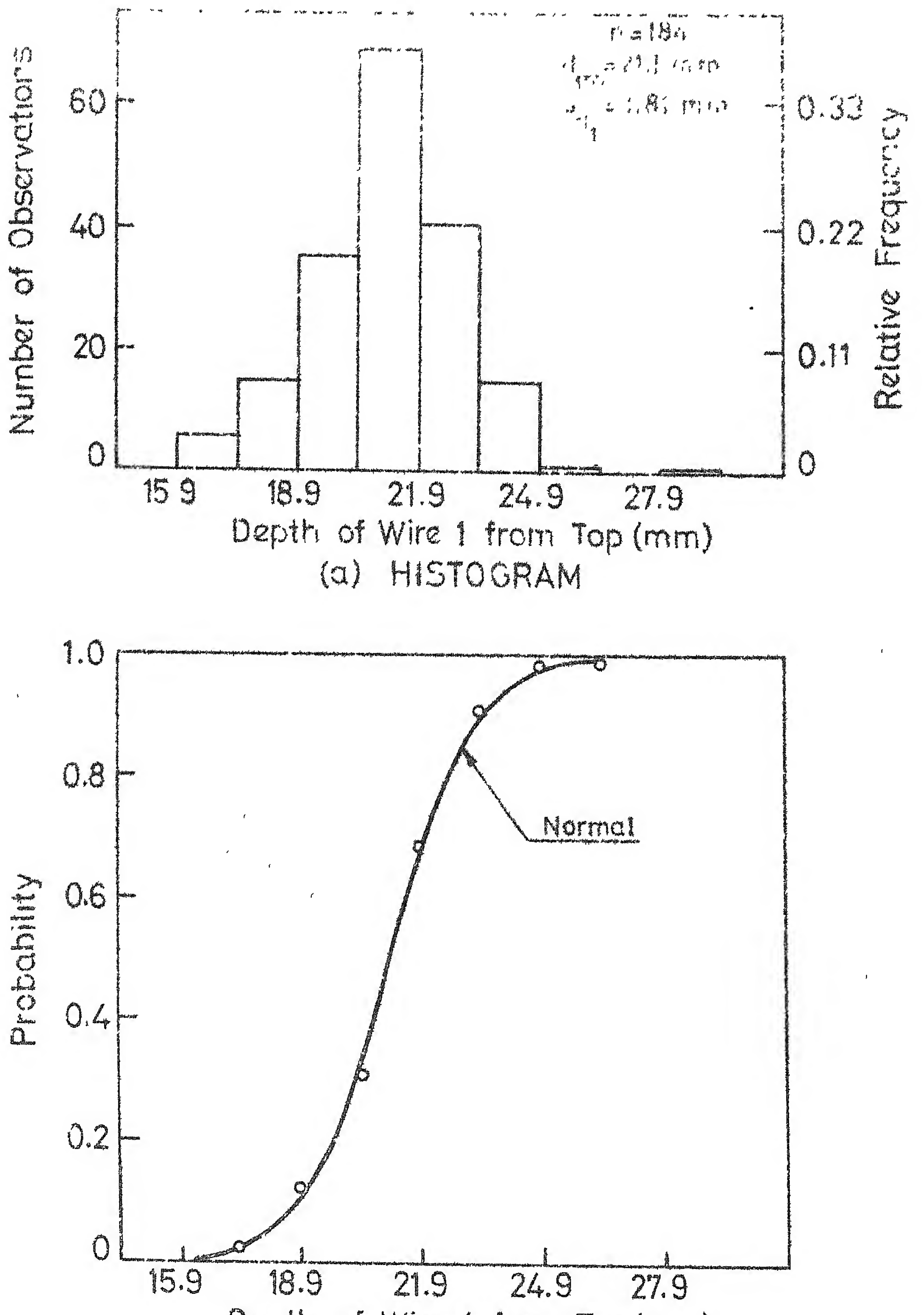
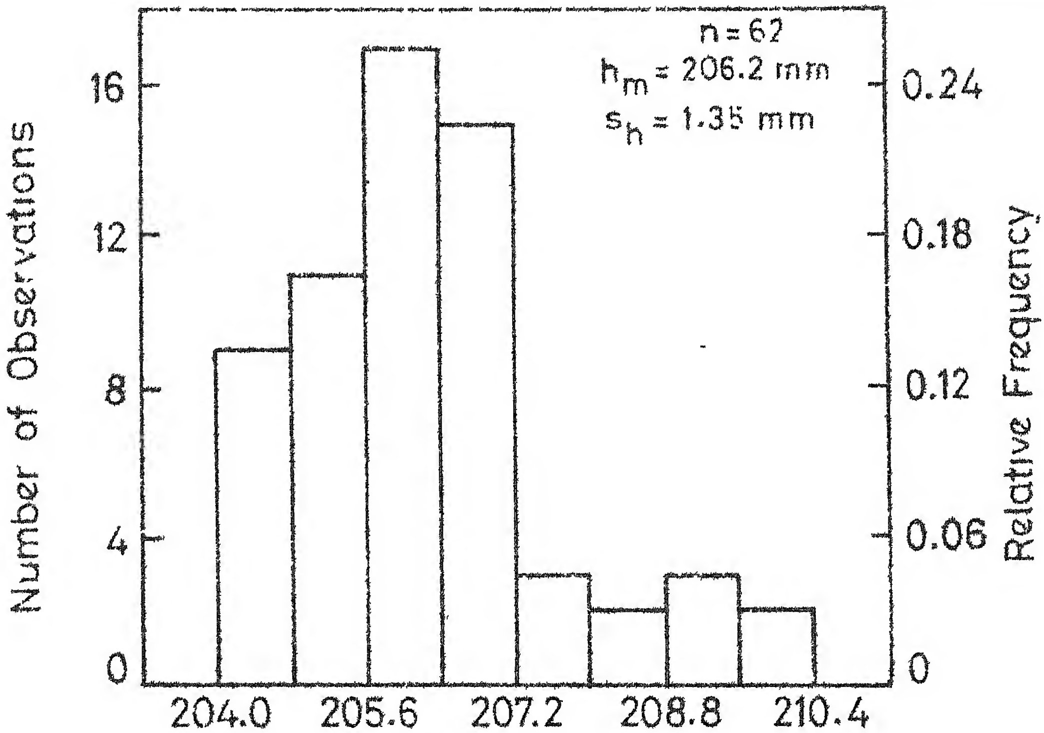
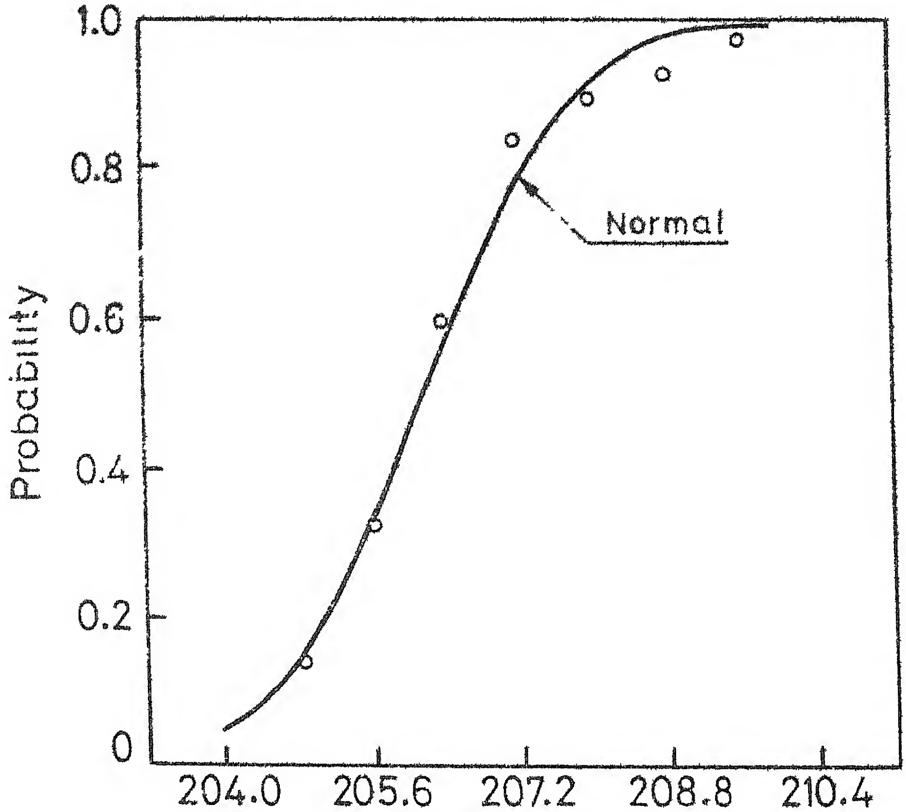


FIG. 2.23 VARIATION OF DEPTH OF WIRE, d_1, OF THE SECTION SHOWN IN FIG. 2.19



(a) HISTOGRAM



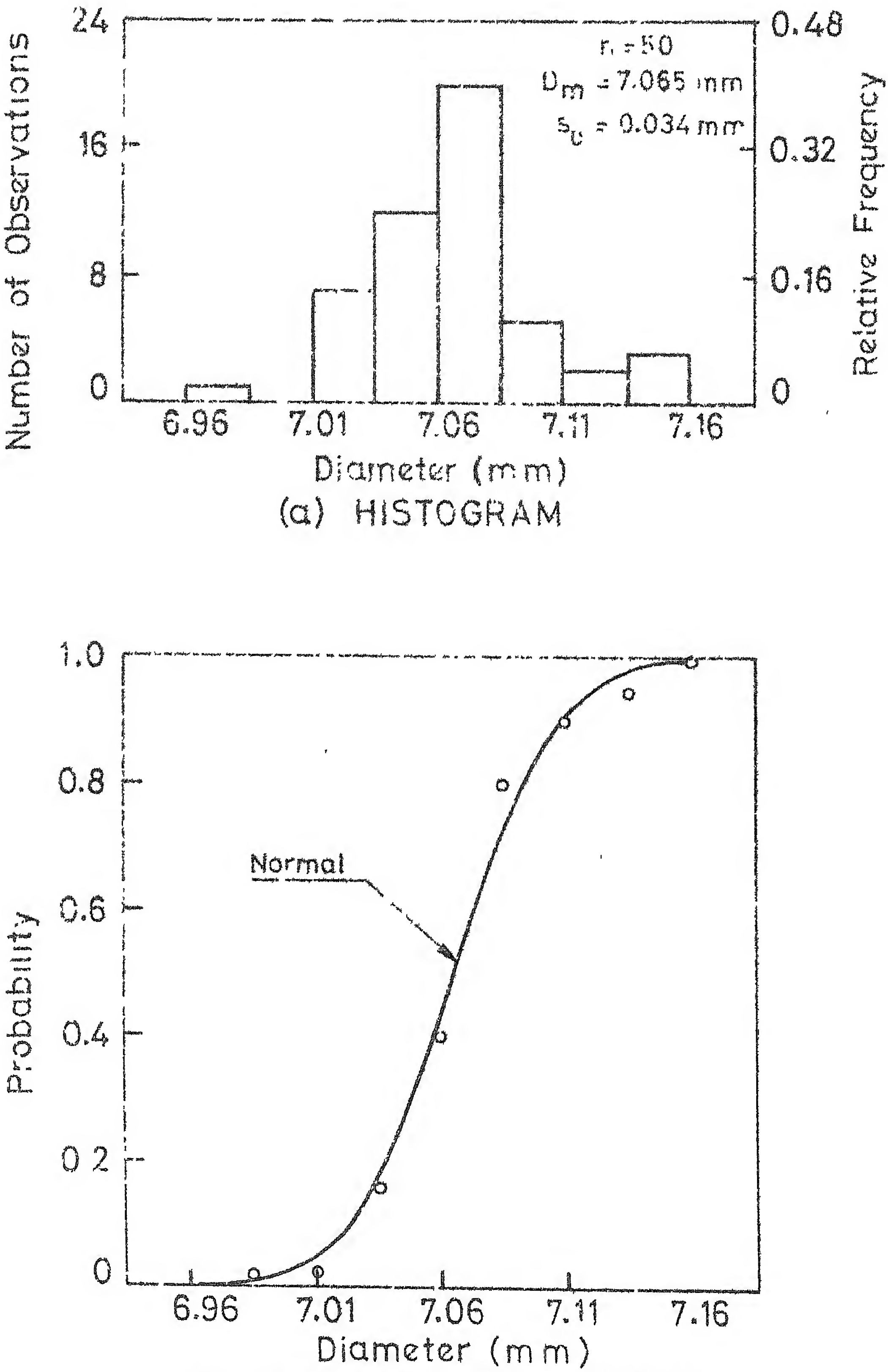
(b) CUMULATIVE DISTRIBUTION

FIG.2.24 VARIATION OF TOTAL DEPTH OF THE SECTION SHOWN IN FIG. 2.19

Table 2.6. Values of specified, mean, standard deviation, and the ratio of mean value to its specified value of geometric properties of the section of Fig. 2.19.

Sl. No.	para-meter	Specified Value cm	Mean Value cm	Standard Deviation cm	Coeff. of vari-ation	Mean Value / Specified Value
1.	b_t	20	20.63	0.164	0.00794	1.031
2.	b'	14	13.93	0.128	0.00918	0.995
3.	t_1	2.5	2.783	0.152	0.0546	1.113
4.	t_2	4	4.126	0.193	0.0468	1.031
5.	d_1	2	2.11	0.181	0.0858	1.055
6.	d_2	13	15.3	0.244	0.0159	1.176
7.	d_3	16.3	16.59	0.226	0.0136	1.017
8.	h	20	20.62	0.135	0.0065	1.031

the probability distribution of diameter of steel, D_s, is required. Fig. 2.25 represents the histogram and cumulative distribution of the diameter of 7mm ϕ HTS wire. Normal distribution is found to satisfy the chi square test at one percent level of significance. The coefficient of variation is observed to be very small.



(a) HISTOGRAM

(b) CUMULATIVE DISTRIBUTION

FIG.2.25 VARIATION OF DIAMETER OF 7 mm ϕ HTS WIRE

CHAPTER 3

RELIABILITY ANALYSIS OF PRESTRESSED CONCRETE BEAMS AT STRENGTH LIMIT STATE UNDER DESIGN LOAD

3.1 GENERAL

The probability distributions which have been determined in Chapter 2 for the field data on the strengths of M350 concrete and high tensile steel (7mm ϕ), load and geometric properties of the seotion, are used in this chapter for the analysis of probability of failure of prestressed concrete (PSC) beams. A method of formulation for the reliability analysis of PSC beams at limit of state of strength under design load, using Monte Carlo technique, is presented for deterministic and probabilistic loads considering probabilistic variations (PV) of

(i) strengths of concrete and steel and

(ii) strengths of concrete and steel and geometric properties of the section.

3.2 INTRODUCTION

Generally PSC structures have symmetrical or unsymmetrical I or T sections. Putcha Chandrasekar (44) analysed the sections based on the governing equation for the ultimate **resisting moment**, M_r, of a section fixed by the deterministic analysis and assumed normal distribution for the resistance of the section. However, in I and T sections, because of the random variations of the

parameters of the resistance of a section, the failure can take place with anyone of the following events:

Y_1 - the section is under-reinforced with the neutral axis in the flange

Y_2 - the section is under-reinforced with the neutral axis in the web

Y_3 - the section is over-reinforced with the neutral axis in the flange

Y_4 - the section is over-reinforced with the neutral axis in the web.

The above events are assumed as mutually exclusive. The occurrence of each event has a certain probability. The probability tree for failure of a section at limit state of strength under design load is given in Fig. 3.1. It can be seen that the probability of failure, p_f, of a section is the sum of the conditional probabilities of failures of the section under each given event and the same can be expressed as

$$p_f = \sum_{i=1}^{4} P(F|Y_i)\ P(Y_i) \tag{3.1}$$

where $P(F|Y_i)$ denotes the conditional probability of F for given event Y_i. F denotes the event 'failure'. $P(Y_i)$ is read as the probability of the event Y_i. The conditional probability of failure of a section for any given event (say Y_i) is given by

$$P(F|Y_i) = P[(R-S) < 0\ |\ Y_i] \tag{3.2a}$$

or,

$$P(F|Y_i) = P[(\frac{R}{S} < 1)\ |\ Y_i] \tag{3.2b}$$

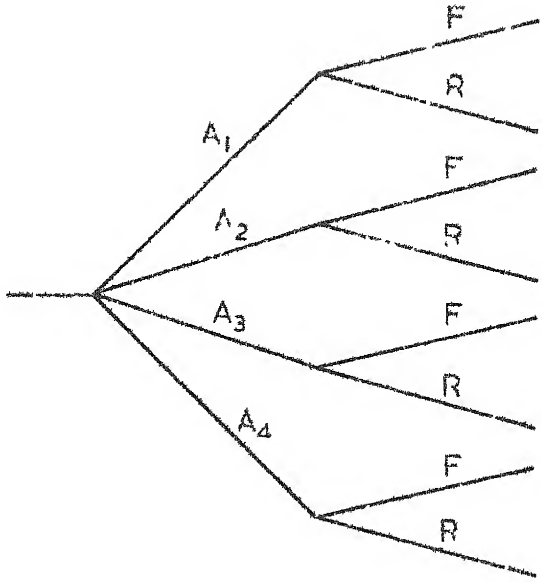


FIG.3.1 PROBABILITY TREE DIAGRAM

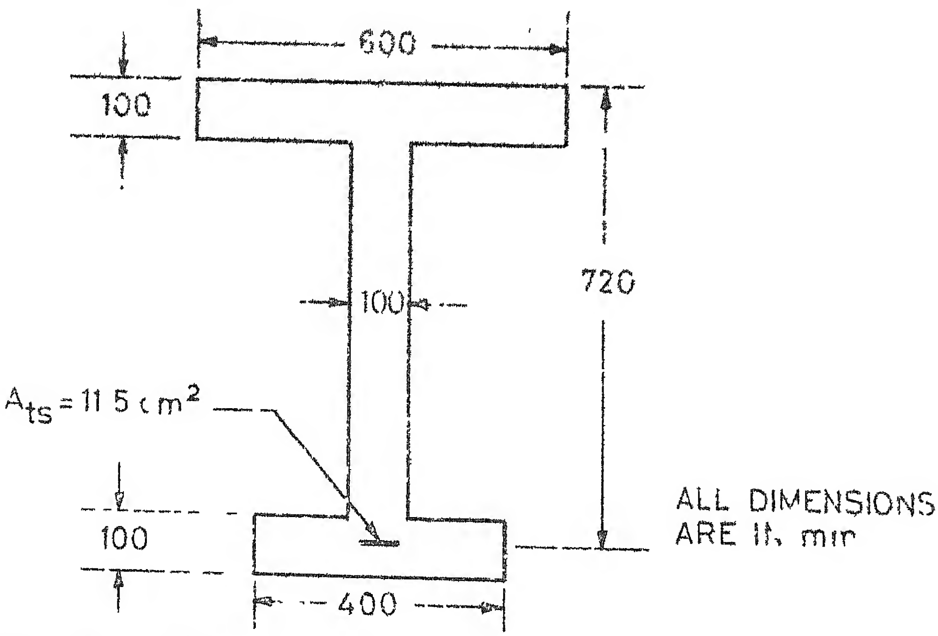


FIG.3.2 TYPICAL PRESTRESSED CONCRETE SECTION (IDEALISED)

where R is the resistance of the section and S is the action (load or bending moment) on the section. The resistance of the section is a function of various material and geometric properties of the section.

$$R = f(X_1, X_2, \ldots\ldots X_n) \tag{3.3}$$

Because parameters X_j are usually random variables, the resistance is also a random variable with density function f_R and cumulative distribution F_R. Assuming the X_j in Eq. 3.3 are statistically independent, their joint density function is

$$f(X_1, X_2, \ldots\ldots X_n) = \prod_{j=1}^{n} f_{X_j}(x_j) \tag{3.4}$$

and its cumulative probability is

$$F_R(r) = P(R \leq r) = \int_G \ldots \int \prod_{j=1}^{n} f_{X_j}(x_j)\, dx_j \tag{3.5}$$

The restriction $R \leq r$ defines the region of integration G in Eq. 3.5. The integral contained in Eq. 3.5 cannot be evaluated in closed form. Added to this, the evaluation of p_f requires the evaluation of the probability of the occurrence of each given event A_i. Defining

B_1 = the event that the section is under-reinforced

B_2 = the event that the section is over-reinforced

B_3 = the event that the neutral axis lies in the flange

B_4 = the event that the neutral axis lies in the web

the probability of the occurrence of the event Y_1 is

$$P(Y_1) = P(B_1 \cap B_3) \tag{3.6}$$

The events B_1 and B_3 are dependent on each other and the density function of each is again the function of material and geometric properties of the section. Hence the evaluation of Eq. 3.6 is again difficult. Finally, to calculate the conditional probability of failure for the given event, Eq.3.2a or 3.2b is to be used which involves numerical integration. The evaluation of structural safety can thus become a formidable task, even when adequate statistical data are available.

3.3 MONTE CARLO TECHNIQUE

One of the usual objectives in using the Monte Carlo simulation technique is to estimate certain parameters and probability distributions of random variables whose values depend on the interactions with random variables whose probability distributions are specified. As it is known that the ultimate resisting moment of a section is a function of several random variables, the probability distribution of M_r depends on the equation connecting these random variables. As explained in the previous section, as closed form solution to calculate cumulative probability of M_r is not possible, Monte Carlo method has been used in this thesis. Secondly, as seen in the previous section, the failure of a flanged section can take place under different events. Hence to study

and simulate the complete random behaviour of the section at limit state of strength, Monte Carlo technique is the best suited method and used in this thesis to obtain the probability distribution and parameters of M_r. This method consists of the following three steps (16):

1) Generating a set of values x_{jk} for the material properties and geometric parameters X_j in accordance with the empirically determined or assumed density functions f_{X_j},
2) Calculating the value of r_k corresponding to the set of values x_{jk} obtained in step 1, by means of the appropriate resistance equation 3.3,
3) Repeating steps 1 and 2 to obtain a large sample values of R and therefore estimating $F_R(r)$.

The parameters for drawing kth set of input values x_{jk} from the corresponding density functions f_{X_j} is to generate first a sequence of n random numbers r_{jk} with uniform density in the range $0 \leq r \leq 1.0$. These generated random numbers are then transformed to their corresponding values of the particular distribution with its parameters. Number of methods are available to generate uniformly distributed random numbers (54).

Assuming σ_{cu} and σ_s are distributed as N(422.8,56) kg/cm^2 and N(15680, 488) kg/cm^2 respectively, samples have been generated for M_r, using Eq.3.13, of the section shown in Fig. 3.2, by Monto Carlo

method. $N(X_1, X_2)$ denotes normal distribution with parameters X_1 and X_2. Figs. 3.3a, 3.3b and 3.3c show the plot of generated random samples of σ_{cu}, σ_s and M_r.

3.4 SAMPLE SIZE

The generated data is used for estimating mean and standard deviation of the resistance of a section. As larger and larger samples are used, the estimates are closer to the population values. The minimum size of the sample depends on the desired accuracy of estimates.

For the estimate of the population mean of a random variabl X, the minimum sample size is specified (54) such that the probabil of the true mean falling within the confidence interval

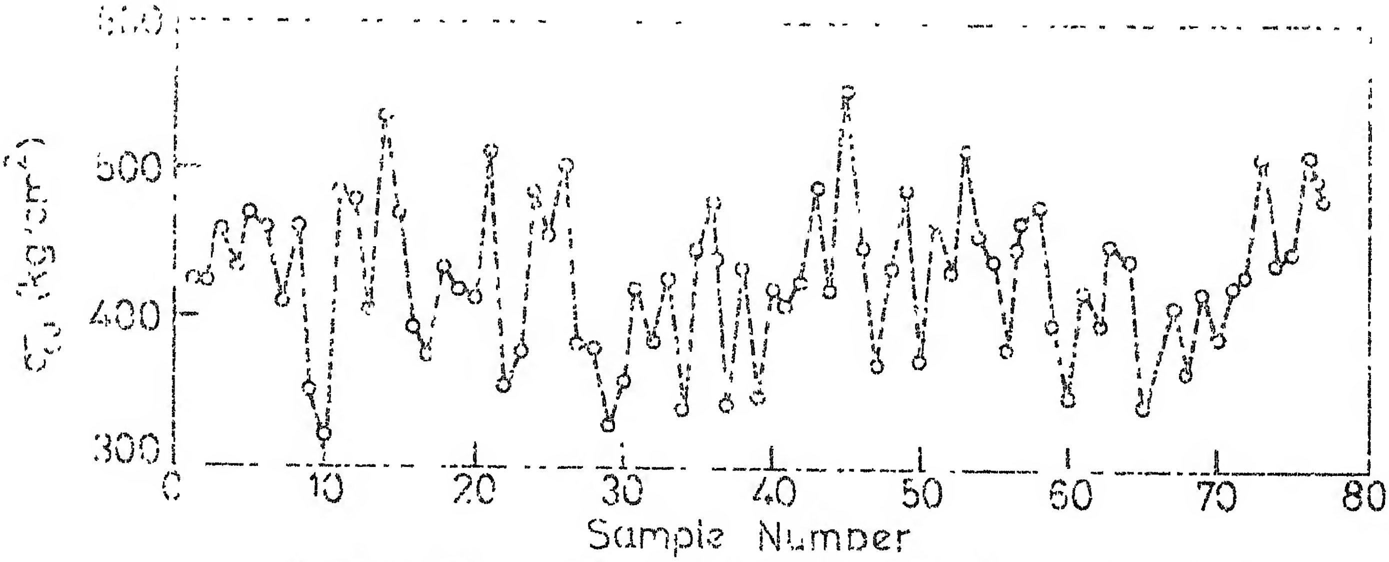
$$\bar{X}_m \pm \phi_{\alpha/2} \left(\frac{\bar{s}_x}{\sqrt{n}}\right) \tag{3.7}$$

is (1-α) percent where $\bar{X}_m$ and $\bar{s}_x$ are sample mean and standard deviation of X . If the length of the confidence interval, e_m, i. acceptable error in the estimate of mean value of X , is specified as

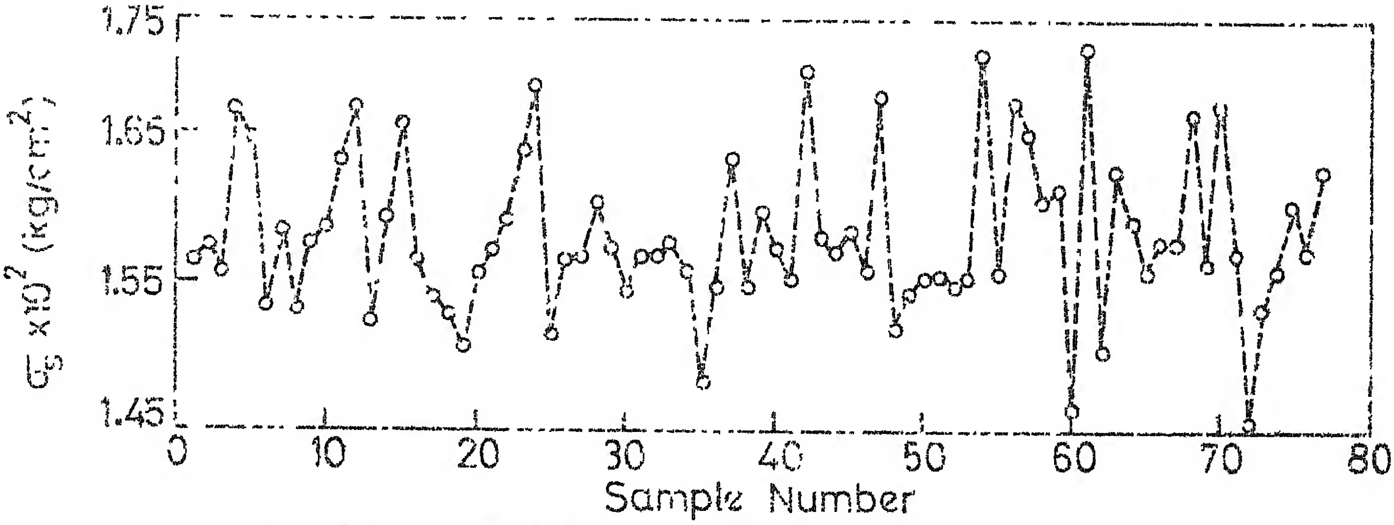
$$e_m = \phi_{\alpha/2} \left(\frac{\bar{s}_x}{\sqrt{n}}\right) \tag{3.8}$$

then minimum size of sample for the estimate of the population of X is given by

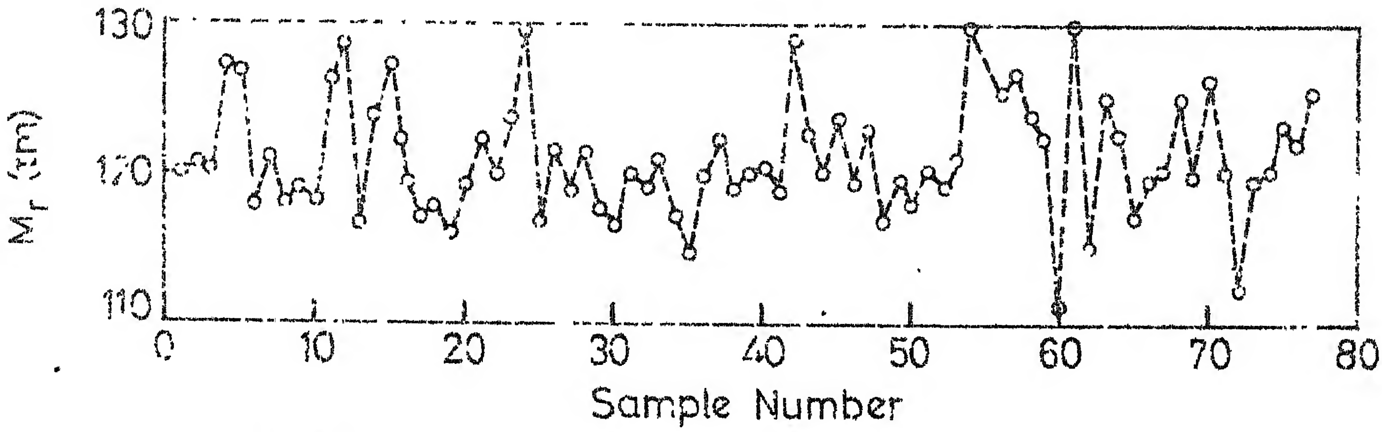
$$n = \phi^2_{\alpha/2} \left(\frac{\bar{s}_x}{e_m}\right)^2 \tag{3.9}$$



(a) GENERATED RANDOM SAMPLES FOR σ_{cu} -N(422.8, 56) kg/cm^2



(b) GENERATED RANDOM SAMPLES FOR σ_s -N(15680, 488) kg/cm^2



(c) SAMPLES OF M_r USING GENERATED SAMPLES OF σ_{cu} AND σ_s

FIG. 3.3. GENERATED RANDOM SAMPLES

For large sample size (say $n > 120$), the standard deviation of standard deviation of x is equal to $\frac{\bar{s}_x}{\sqrt{2n}}$. Hence for the estimate of standard deviation of X, the minimum size is specified such that the probability of true standard deviation falling within the confidence interval (55)

$$\bar{s}_x \pm \phi_{\alpha/2} \left(\frac{\bar{s}_x}{\sqrt{2n}}\right) \tag{3.10}$$

is $(1-\alpha)$ percent. Specifying,

$$e_s = \phi_{\alpha/2} \left(\frac{\bar{s}_x}{\sqrt{2n}}\right) \tag{3.11}$$

the minimum sample size for the estimate of the population standard deviation of X is given by

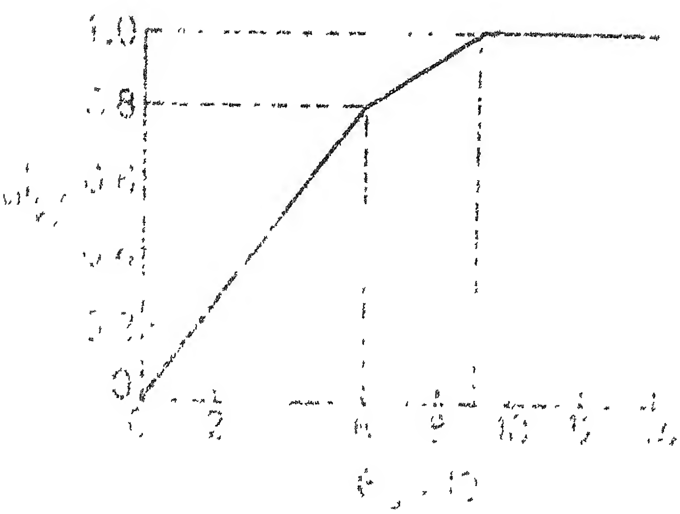
$$n = \frac{1}{2} \phi^2_{\alpha/2} \left(\frac{\bar{s}_x}{e_s}\right)^2 \tag{3.12}$$

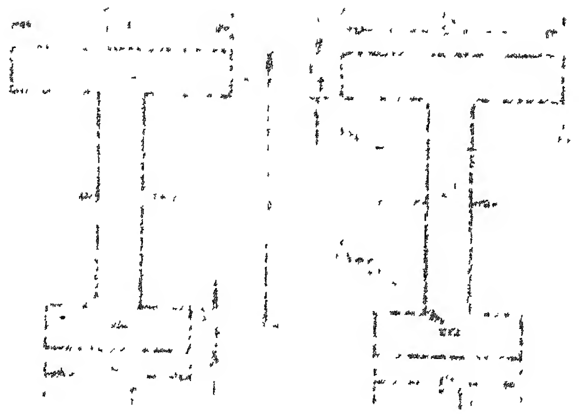
where e_s is the acceptable error in the estimate of standard deviation of X.

In all examples in this thesis more samples, than required by Eqs. 3.9 and 3.12 for $\alpha = 5\ \%$ and acceptable error of 5 percent in the estimates of mean and standard deviations, are generated.

3.5 EQUATIONS FOR DETERMINATION OF ULTIMATE STRENGTH OF A PSC SECTION

It is assumed that the beams are fully bonded and the stress-strain curve for steel is as shown in Fig. 3.4. Equations, which are necessary for the Monte Carlo method, to determine M_r for each event Y_1, Y_2, Y_3 and Y_4 are given below.





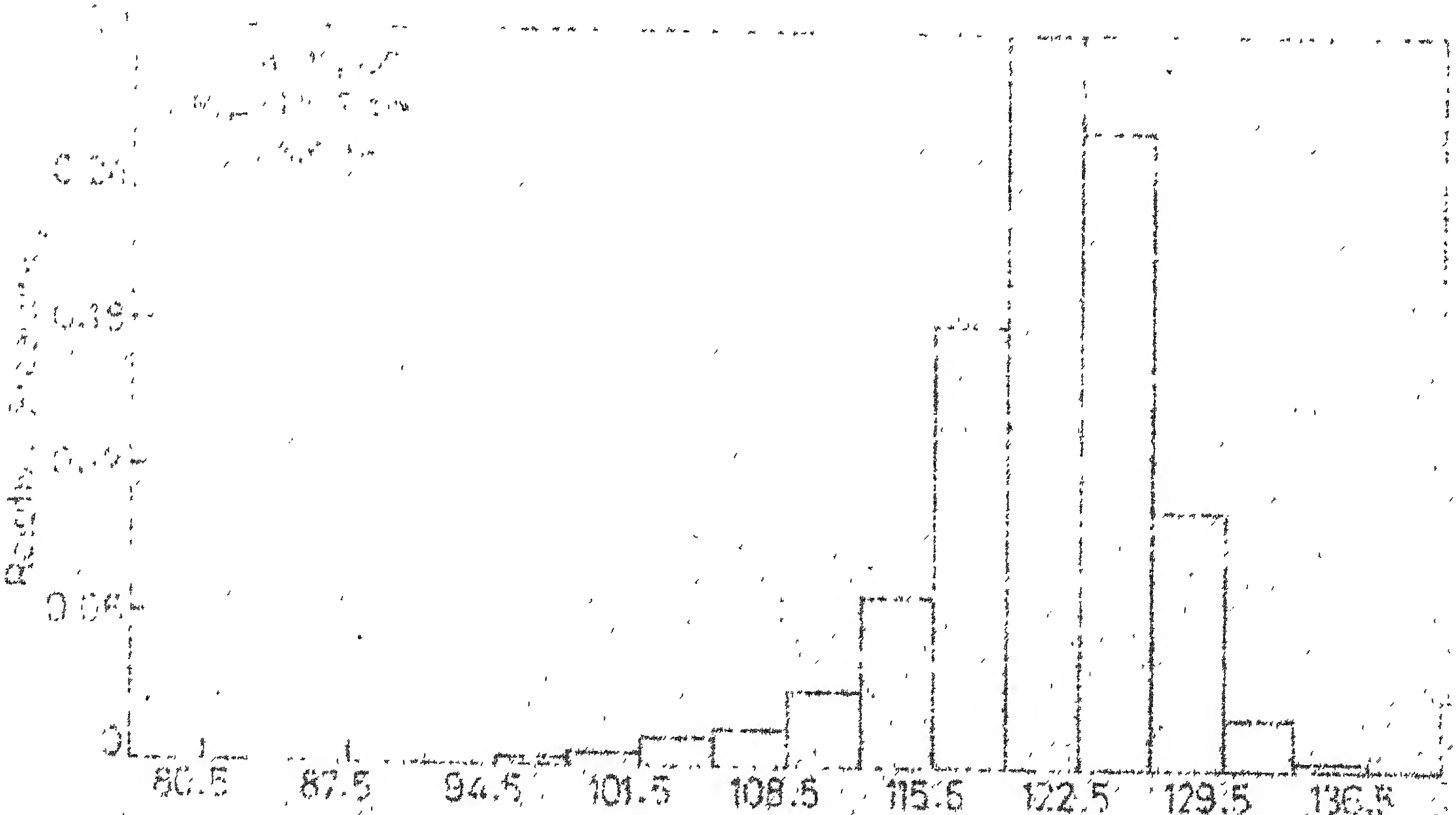


FIG. 3.6 HISTOGRAM OF M_u OF SECTION OF FIG. 3.2 FOR PV OF 9% AND σ

Event Y_1

The ultimate resisting moment of the section, M_{ruf}, for the event Y_1 in which the section is under-reinforced and the neutral axis (NA) lies in the flange (Fig. 3.5a) is given by (46)

$$M_{ruf} = \sigma_s A_{ts} d \left(1 - \frac{0.75\sigma_s A_{ts}}{b_t d \sigma_{cu}}\right) \tag{3.13}$$

where A_{ts} is the area of prestress steel. The section is under-reinforced if

$$\frac{A_{ts}\sigma_s}{b_t d\sigma_{cu}} \leq 0.24 \tag{3.14}$$

Event Y_2(Fig. 3.5b)

The ultimate resisting moment of the section, M_{ruw}, for the event Y_2 in which the section is under-reinforced and the neutral axis lies in the web, is given by (46)

$$M_{ruw} = 0.7\sigma_{cu} (b_t-b') t_t(d-0.5 t_t)$$

$$+ \sigma_s A_{tsw} d \left(1 - \frac{0.75 A_{tsw} \sigma_s}{b'd \sigma_{cu}}\right) \tag{3.15}$$

where

$$A_{tsw} = A_{ts} - A_{tsf} \tag{3.16}$$

and $$A_{tsf} = \frac{1}{\sigma_s} [0.68 \sigma_{cu}(b_t-b') t_t] \tag{3.17}$$

The Eq. 3.15 is valid when

$$\frac{A_{tsw} \sigma_s}{b'd \sigma_{cu}} \leq 0.24 \tag{3.18}$$

Event Y_3

For the event Y_3, i.e. the neutral axis lies in the flange and the section is over-reinforced, the stress in steel, σ_{su}, at the time of failure of the section is given by

$$\sigma_{su} = \frac{1}{2} (b_1 + \sqrt{b_1^2 + 4a_1}) \tag{3.19}$$

where

$$b_1 = \frac{\sigma_s}{0.015} (0.006 + \varepsilon_c - \varepsilon_{sp} - \varepsilon_{ce})$$

$$a_1 = \frac{\sigma_s}{0.015\ A_{ts}} (0.68\ \sigma_{cu}\ b_t\ 0.8\ d\ \varepsilon_c)$$

in which ε_c is the limiting compressive strain in concrete and ε_{sp} is the strain in prestressing steel and ε_{ce} is the strain in concrete due to effective prestress. Assuming the depth of stress block is equal to 0.8 times the depth of neutral axis (45), the depth of stress block, a, is calculated from the formula (56)

$$a = 0.8d \left(\frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s - \varepsilon_{sp} - \varepsilon_{ce}}\right) \tag{3.20}$$

where ε_s is the strain in steel at the time of failure of the section. Assuming the stress-strain curve for steel shown in Fig. 3.4, the value of ε_s is calculated from the equation

$$\varepsilon_s = 0.006 + \frac{0.015}{\sigma_s} (\sigma_{su} - 0.8\ \sigma_s) \tag{3.21}$$

The ultimate resisting moment of the section, M_{rof}, for the event Y_3 is given by

$$M_{rof} = 0.68\,\sigma_{cu}\, b_t\, a\,(d - 0.5a) \qquad (3.22)$$

Event Y_4

For the event Y_4, i.e. the neutral axis lies in the flange and the section is over-reinforced, the value of σ_{su} is given by

$$\sigma_{su} = 0.5\,\left(b_2 + \sqrt{b_2^2 + 4a_2}\right) \qquad (3.23)$$

where

$$b_2 = \frac{1}{A_{ts}}\,[\,0.68\,\sigma_{cu}\,(b_t - b')\,t_t\,]$$

$$+ \frac{\sigma_s}{0.015}\,(0.006 + \varepsilon_c - \varepsilon_{sp} - \varepsilon_{ce})$$

$$a_2 = \frac{0.68\,\sigma_{cu}\,\sigma_s}{0.015\,A_{ts}}\,[0.8b'd\,\varepsilon_c - (b_t - b')t_t(0.006 + \varepsilon_c - \varepsilon_{sp} - \varepsilon_{ce})\,]$$

After evaluating σ_{su}, the determination of ε_s and 'a' are same as explained for event Y_3. Values of 'a' and ε_s are given by Eqs.3.20 and 3.21 respectively. The ultimate resisting moment of the section M_{row}, for the event Y_4, is given by

$$M_{row} = 0.68\,\sigma_{cu}\,[(b_t - b')t_t(d - 0.5t_t) + b'a(d - 0.5a)] \qquad (3.24)$$

In every case, it can be seen that the resistance is a function of strengths of concrete and steel and geometric properties of the section.

3.6 PROBABILITY DISTRIBUTION OF M_r OF A SECTION FOR PROBABILISTIC VARIATIONS OF STRENGTHS OF MATERIALS

Even though all parameters in the equations for determination of M_r of a section are random variables, σ_{cu} and σ_s are

first considered and the reliability analysis presented. The typical prestressed concrete section shown in Fig. 3.2 has been used for the purpose of study of the probability distribution of M_r. With strengths of concrete N(422.8, 56) kg/cm^2 and steel N(15680,488) kg/cm^2 Monte Carlo technique has been used and samples have been generated using Eqs. 3.13 to 3.21 for M_r of the section. Different sample sizes have been considered from 500 to 10,000. The effect of sample size and minimum sample size required for the determination of parameters of a random variable have already been given in article 3.4. Fig. 3.6 shows the histogram of the generated data for M_r of the section shown in Fig. 3.2 for a sample size of 10,000. It is seen that the distribution is negatively skewed. Normal, beta, Type I extremal (smallest) and Type III extremal (smallest) distributions have not been found to satisfy the chi square test for the generated data. The samples, for which the section is under-reinforced and over-reinforced have been arranged separately and histograms and cumulative distributions of the data are shown in Figs. 3.7 and 3.8. With the deterministic analysis the section is under-reinforced ($\frac{A_{tsw}\,\sigma_s}{b'd\sigma_{cu}} = 0.2$) for $\sigma_{cu} = 350$ kg/cm^2 and $\sigma_s = 15000$ kg/cm^2. It is observed from Fig. 3.8a that about 10 percent of samples lie in the over-reinforced case. Hence there is a probability of the section becoming over-reinforced even though the section is under-reinforced deterministically. It is found that the normal distribution satisfies the generated data (under-reinforced case) shown in

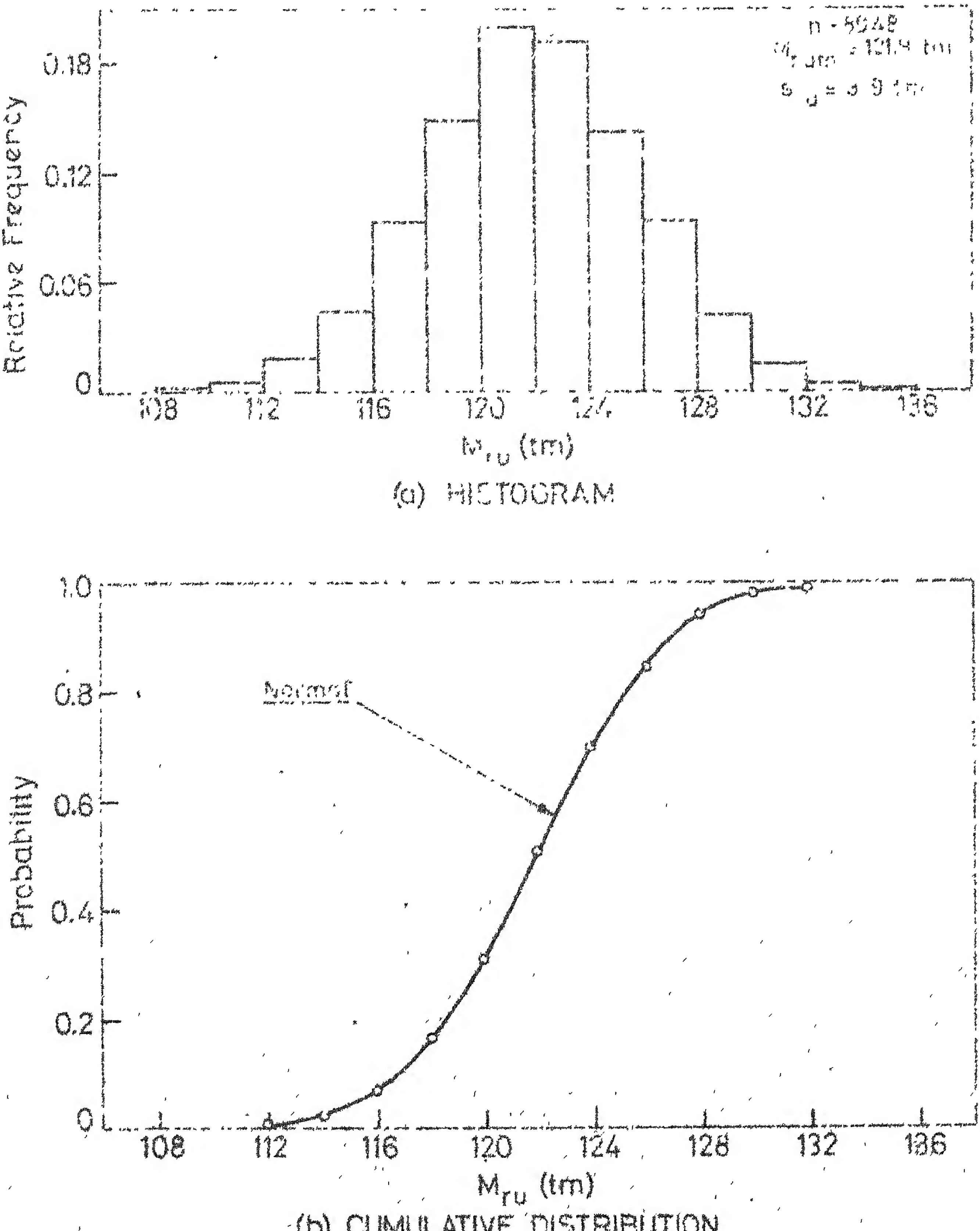


FIG. 3.7 VARIATION OF M_{ru} OF SECTION OF FIG. 3.2 FOR P V OF σ_{cu} AND σ_s

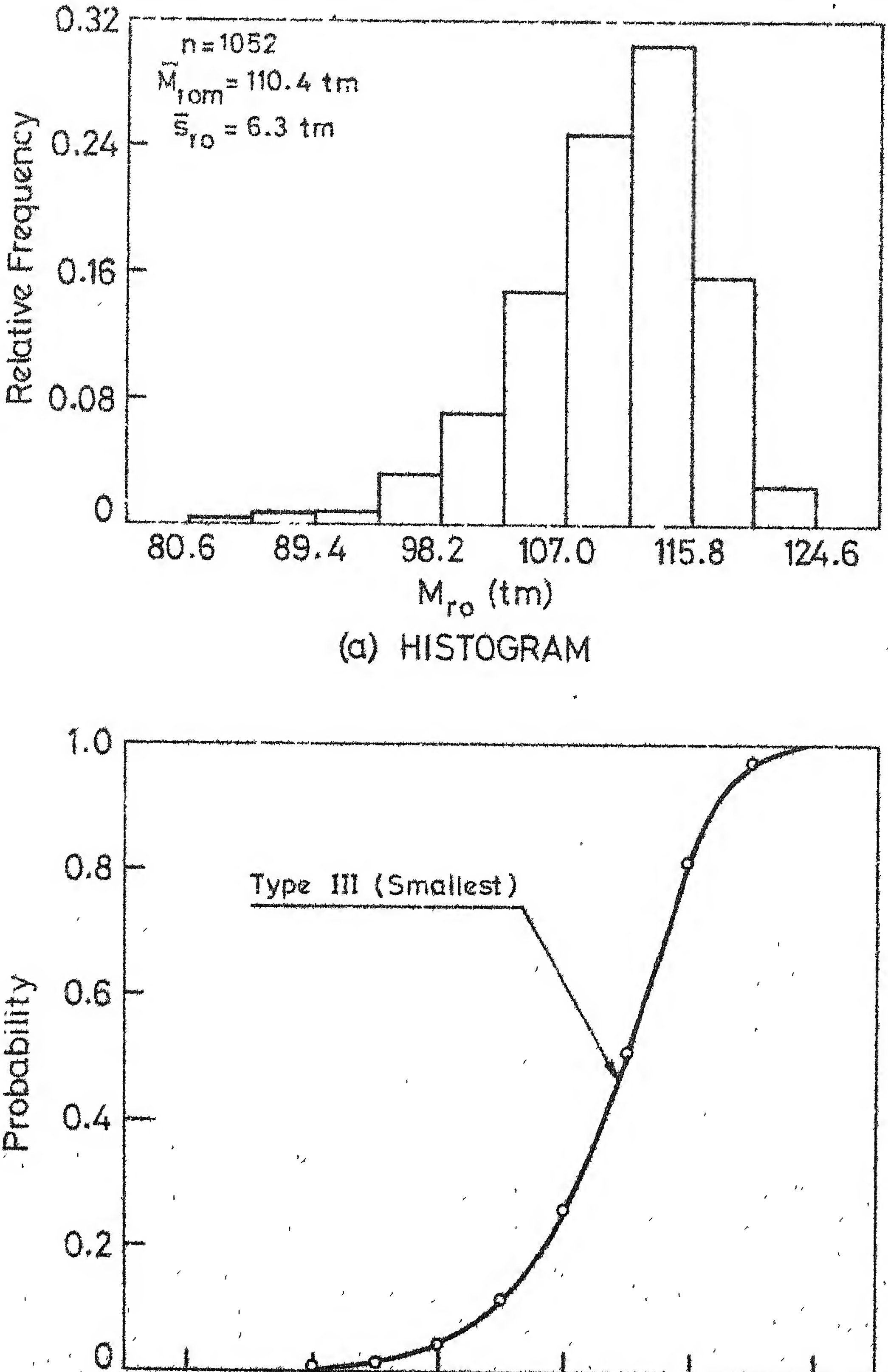


FIG.3.8 VARIATION OF M_{ro} OF SECTION OF FIG.3.2 FOR P.V OF σ_{cu} AND σ_s

Fig. 3.7a at one percent level of significance. However, normal distribution does not satisfy the generated data (over-reinforced case) shown in Fig. 3.8a and it is found that Type III extremal (smallest) distribution satisfies the same data at one percent level of significance. Hence normal and Type III extermal (smallest) distributions are adopted for ultimate resisting moments of the section for under-reinforced and over-reinforced cases respectively.

The probability density function of the normal distribution of M_{ru}, the ultimate resisting moment of the section for under-reinforced case, is given by

$$f_{M_{ru}}(M_{ru}) = \frac{1}{\sqrt{2\pi}\, s_{ru}} \exp\left[-\frac{1}{2}\left(\frac{M_{ru}-M_{rum}}{s_{ru}}\right)^2\right] \tag{3.25}$$

where M_{rum} and s_{ru} are parameters mean and standard deviation of the resisting moment of the section for under-reinforced case. The cumulative distribution of M_{ru} is written as

$$F_{M_{ru}}(M_{ru}) = \phi\left(\frac{M_{ru}-M_{rum}}{s_{ru}}\right) \tag{3.26}$$

The probability density function of the Type III extremal (smallest) distribution of M_{ro} the ultimate resisting moment of the section for the over-reinforced case, is given by (48),

$$f_{M_{ro}}(M_{ro}) = \frac{s_{ro}}{M_{rom}}\left(\frac{M_{ro}}{M_{rom}}\right)^{s_{ro}-1} \exp\left[-\left(\frac{M_{ro}}{M_{rom}}\right)^{s_{ro}}\right] \qquad M_{ro} \geq 0 \tag{3.27}$$

where M_{rom} and s_{ro} are parameters of the distribution of M_{ro}.

The parameters are estimated (48) as follows.

$$M_{rom} = \frac{\bar{M}_{rom}}{\Gamma(1 + s_{ro})} \tag{3.28}$$

$$\delta_{ro}^2 = \frac{\Gamma(1 + \frac{2}{s_{ro}})}{\Gamma^2(1 + \frac{1}{s_{ro}})} - 1 \tag{3.29}$$

where $\bar{M}_{rom}$ and δ_{ro} are the mean and coefficient of variation of M_{ro}. The relationship of Eq. 3.29 is graphed in Fig. 3.9, permitting one to find the parameter s_{ro} if δ_{ro} is known for the data. The cumulative distribution of M_{ro} is given by

$$F_{M_{ro}}(M_{ro}) = 1 - \exp\left[-\left(\frac{M_{ro}}{M_{rom}}\right)^{s_{ro}}\right] \qquad M_{ro} \geq 0 \tag{3.30}$$

3.7 RELIABILITY ANALYSIS FORMULATION OF A SECTION

Having arrived at the distributions for the ultimate resisting moment of a section for under-reinforced and over-reinforced cases, the reliability analysis is formulated as given below.

The probability of failure of a section is given by,

$$p_f = P(F|U)\,P(U) + P(F|O)\,P(O) \tag{3.31}$$

where U and O represent the events that the section is under-reinforced and over-reinforced respectively. For any given section, using Monte Carlo method, number of samples are generated and the sample means and standard deviations of the generated

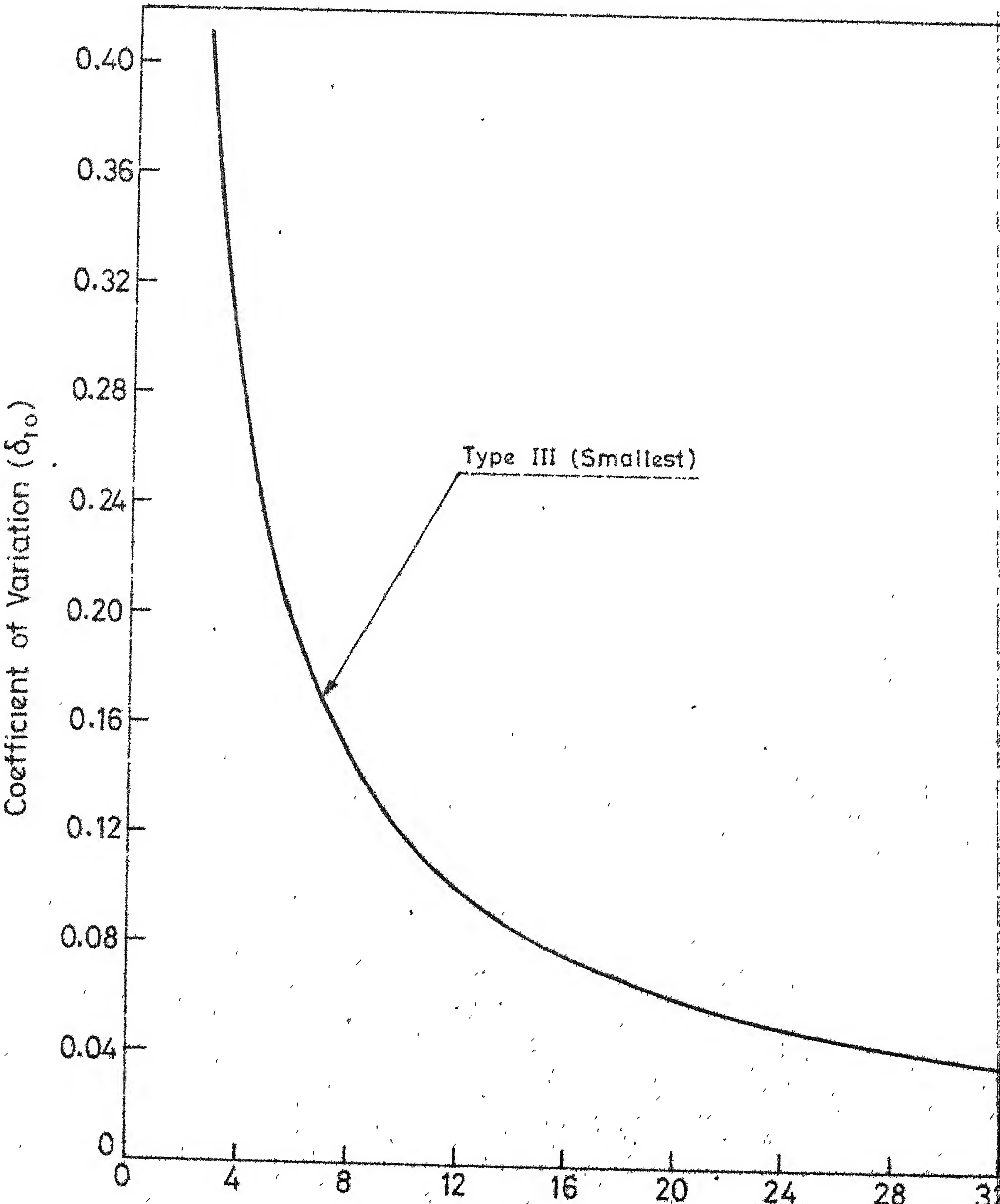


FIG.3.9 EXTREME VALUE DISTRIBUTION - δ_{ro} VERSUS s_{ro}

data for the resistance of the section for under-reinforced and over-reinforced cases are found out. From sample means and standard deviations, the parameters of the distributions of M_{ru} and M_{ro} can be estimated. The value of $P(U)$ has been taken as

$$P(U) = \frac{n_u}{n} \tag{3.32}$$

where n_u is the number of samples of under-reinforced case and n is the total number of generated samples. Hence,

$$P(O) = 1 - P(U) \tag{3.33}$$

3.8 COMPUTATION OF p_f FOR DETERMINISTIC LOAD

Assuming the load is deterministic, the conditional probability of failure of the section, $p_{f|u}$, for the given event U, is given by

$$p_{f|u} = P(M_{ru} < M_e | U) \tag{3.34}$$

where M_e is the external bending moment. The hatched area in Fig. 3.10 is the area under integration for the evaluation of probabilities of failure for deterministic loads. For normally distributed M_{ru},

$$P(F|U) = p_{f|u} = \phi\left(\frac{M_e - M_{rum}}{s_{ru}}\right) \tag{3.35}$$

Similarly, the conditional probability of failure of the section, $p_{f|o}$, for the given event O, using Eq. 3.30, is given by

$$P(F|O) = p_{f|o} = 1 - \exp\left[-\left(\frac{M_e}{M_{rom}}\right)^{s_{ro}}\right] \tag{3.36}$$

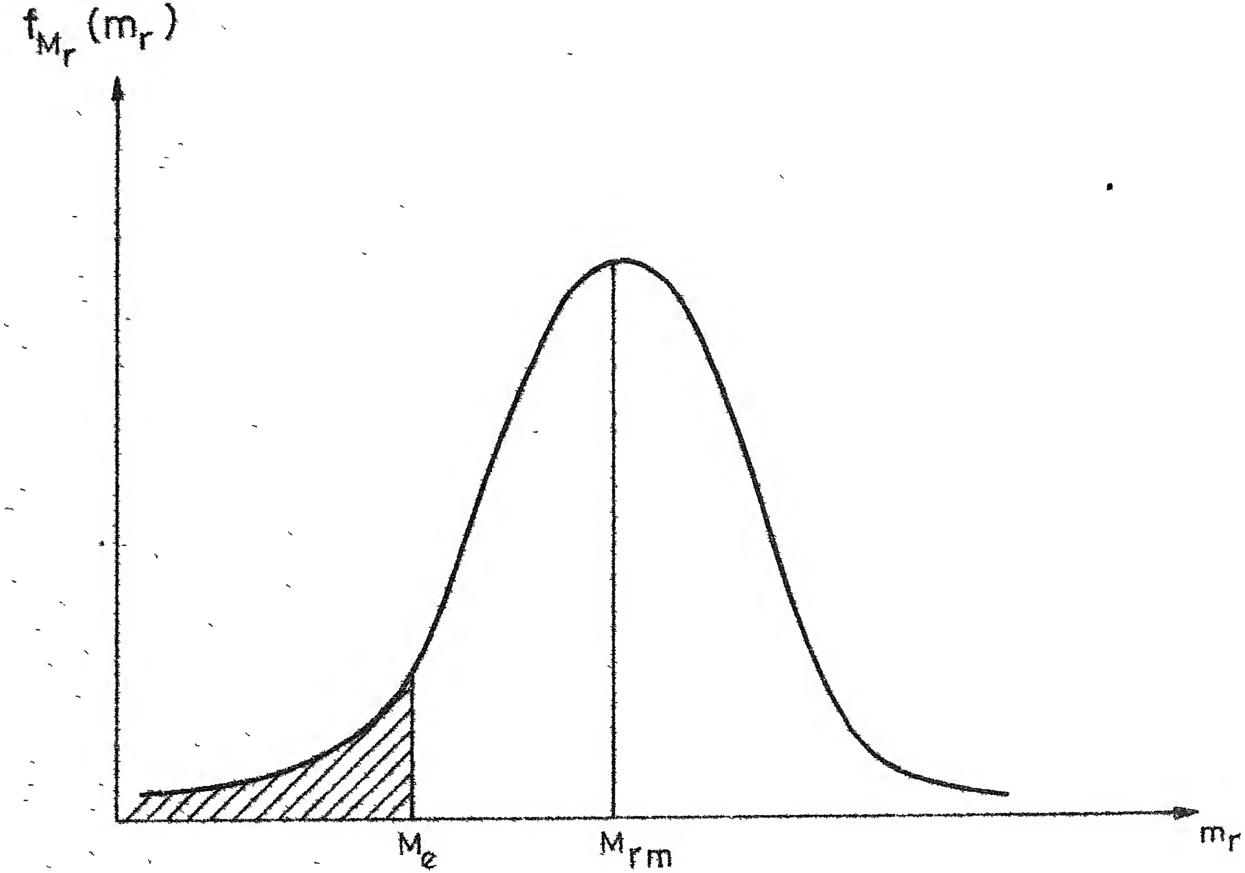


FIG.3.10 FREQUENCY DISTRIBUTION OF MOMENT OF RESISTANCE

The external moment can be working moment, M_{ew}, or ultimate moment, M_{eu}.

$$M_{eu} = F_q\, M_q + F_g\, M_g \tag{3.37}$$

where,

F_q = live load factor

F_g = dead load factor

M_g = working moment due to dead load

M_q = working moment due to live load

The p_f of the section is calculated using Eq. 3.31.

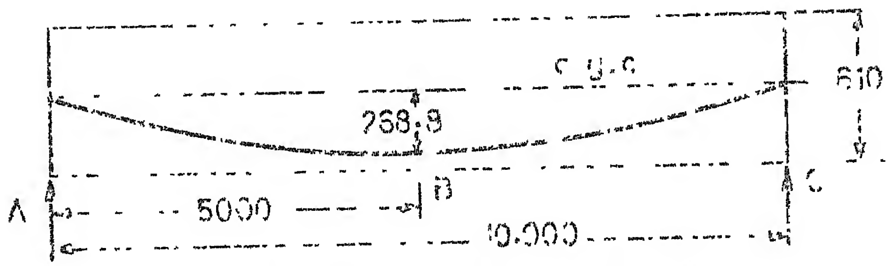
The procedure is illustrated with an example.

Example 3.1

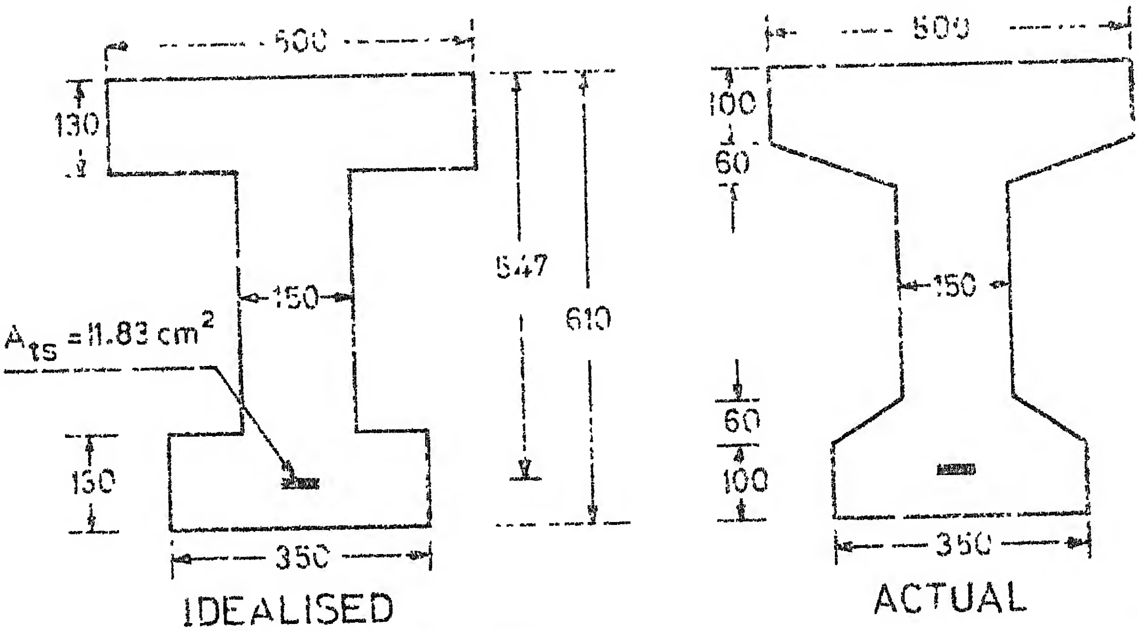
Simply supported PSC beams, supporting a floor, are spaced at 5m. Live load on the floor is 400 kg/m^2. Total dead load (including self weight of the beam) on any intermediate beam is 1500 kg/m. The beams are designed with concrete cube strength of 350 kg/cm^2 and steel strength of 15000 kg/cm^2. The cross section of the beam and cable profile are shown in Fig. 3.11. The effective span and area of steel are 10 m and 11.83cm^2 respectively. The beams are designed assuming they are cast separately and the composite action of slab and beam is not considered. One of the intermediate beams only is considered and the reliability analysis of the same at limit state of strength is illustrated.

Live load on any intermediate beam = 5 $\times$ 400 = 2000 kg/m.

From Chapter 2, for M350 concrete,



(a) PARABOLIC CABLE PROFILE



(b) MIDSPAN SECTION

FIG.3.11 SIMPLY SUPPORTED BEAM-EXAMPLE 3.1

ALL DIMENSIONS ARE IN mm

$$\sigma_{cum} = 422.8 \text{ kg/cm}^2 \; ; \; s_c = 56 \text{ kg/cm}^2$$

$$\sigma_{sm} = 15680 \text{ kg/cm}^2 \; ; \; s_s = 488 \text{ kg/cm}^2$$

using Monte Carlo method, 10000 samples are generated and it is found that number of samples for under-reinforced and over-reinforced cases are equal to 8370 and 1630 respectively. Using Eq. 3.32,

$$P(U) = 0.837 \; ; \; P(O) = 0.163$$

Mean and standard deviation of the generated samples of M_{ru} and M_{ro} are calculated (this is considered as a part of the Monte Carlo method). Their values are given below.

For the under-reinforced case,

$$M_{rum} = 90.57 \text{ tm} \; ; \; s_{ru} = 2.84 \text{ tm}$$

For the over-reinforced case,

$$\bar{M}_{rom} = 82.58 \text{ tm}; \; \bar{s}_{ro} = 4.85 \text{ tm}$$

$$\delta_{ro} = \frac{4.85}{82.58} = 0.0586$$

Using Type III extremal (smallest) distribution for M_{ro}, the parameter s_{ro} is obtained as 21.6 from the Fig. 3.9 for the calculated value of δ_{ro} equal to 0.0586. Using Eq. 3.28, the value of M_{rom} is calculated and found to be equal to 84.8 tm. The value of M_e at the section is 43.75 tm. The conditional probability of failure for the given event U, using Eq. 3.35, is evaluated as

$$p_{f;u} = \phi\left(\frac{43.75 - 90.57}{2.84}\right) < 10^{-24}$$

Hereafter, values less than 10^{-24} will be considered and reported as zero.

The conditional probability of failure for the given event '0', using Eq. 3.36, is calculated as

$$p_{f|o} = 1 - \exp\left[- \left(\frac{43.75}{84.8}\right)^{21.6} \right]$$

$$= 6.44 \times 10^{-6}$$

using Eq. 3.31, the probability of failure of the section is obtained as

$$p_f = (0)\ (0.837) + (6.44 \times 10^{-6})\ (0.163)$$

$$= 1.08 \times 10^{-7}$$

3.9 RELIABILITY ANALYSIS OF CONTINUOUS BEAMS

The critical sections where failures are likely to occur and cause the total collapse of the beam are to be first determined for the analysis of continuous beams. Fixed ends and intermediate supports of the beam, where maximum negative bending moments occur, are the one set of critical sections. The positions of critical sections for positive bending moments depend on the difference between internal resisting moment and external moment at different sections. In the case of probabilistic analysis, it depends on the difference between the mean values of the internal and external moments and the standard deviation of the resisting moment. The resisting moment of the section is approximately proportional to the

eccentricity of the cable. The second order equation for external bending moment for given load is known. The standard deviation of the ultimate resisting moment is approximately equal to the standard deviation of the strength of steel (observed later). Assuming normal distribution for (R-S), it is possible to locate approximately the position of the critical section in each span within a few trials.

After locating critical sections, Monte Carlo method is used and number of samples are generated for the resistance of each critical section. The failure probability of each critical section for each loading case is calculated as explained in article 3.6. In a redundant structure, it is known that the failure of the structure takes place only after sufficient number of sections have failed. In a continuous beam each span has a failure mode under a given load and each critical section has a certain probability of failure. Hence if there are K critical sections in the ith failure mode for given load j, the probability of occurrence of the failure mode, p_{fi}, is given by the probability of intersection of the events Z_k, defining the failure of each critical section. This can be written as

$$\begin{aligned} p_{fij} &= P(\text{Mode i occurs under load j}) \\ &= P(Z_1 \cap Z_2 \cap \cdots\cdots \cap Z_K) \end{aligned} \tag{3.38}$$

Assuming events Z_k are independent,

$$p_{fij} = \prod_{k=1}^{K} p_{fj_k} \tag{3.39}$$

where p_{fz_k} is the probability of failure of the section Z_k. This can be evaluated as explained in article 3.8. If p_{fs_j} is the probability of failure of the system under the load j, then bounds on p_{fs_j} can be defined by (26)

$$\max \; p_{fij} \leq p_{fs_j} \leq \sum_{i=1}^{m} p_{fij} \tag{3.40}$$

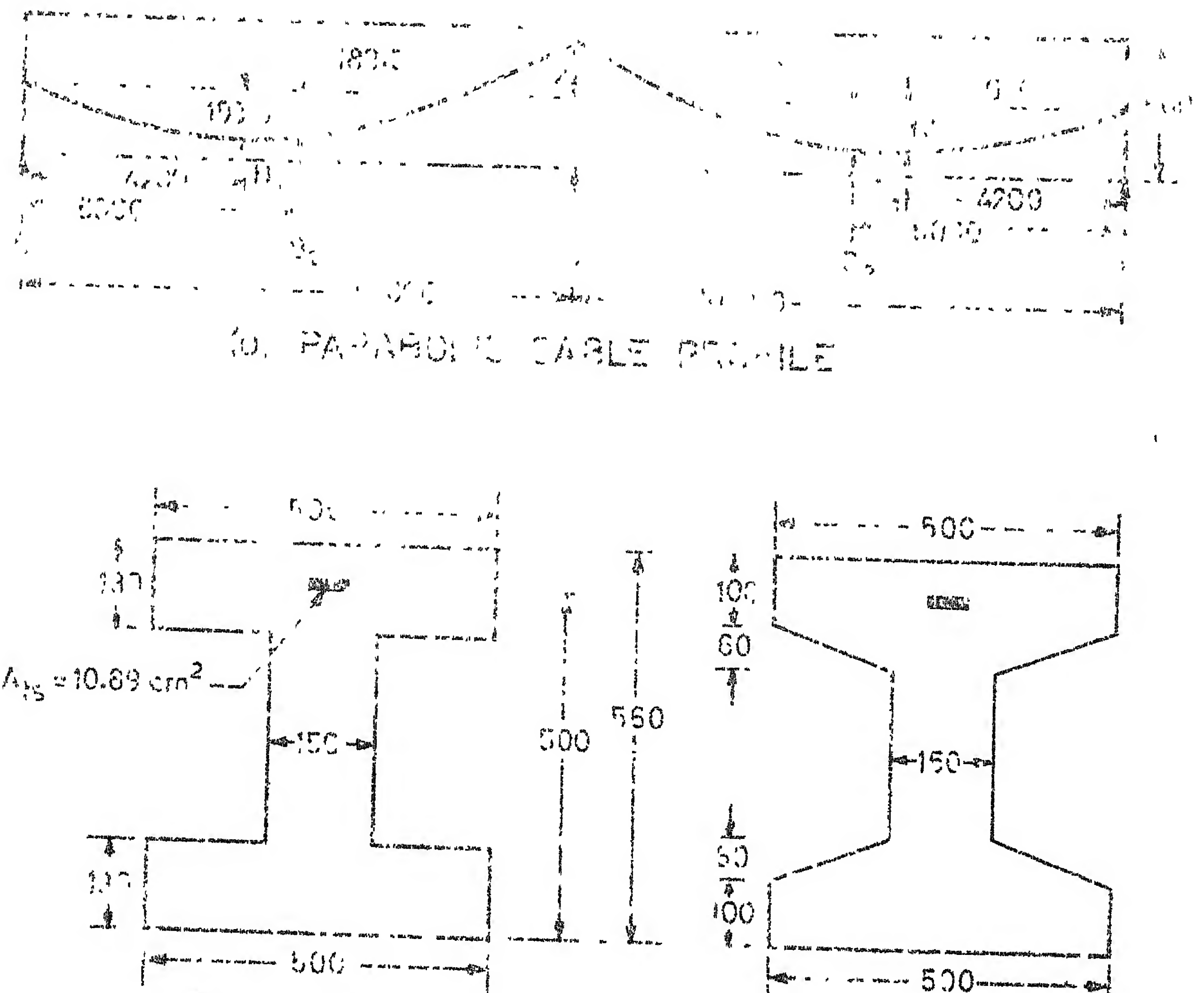
where m is the number of failure modes. Assuming independent load conditions and mode resistances and p_{fs_j} is small compared to 1.0, bounds on p_{fs} are given by (9,26),

$$\max \; p_{fij} \leq p_{fs} \leq \sum_{i=1}^{m} \sum_{j=1}^{n} p_{fij} \tag{3.41}$$

in which p_{fs} is the probability of failure of the system for m modes and n load conditions. The procedure is illustrated with examples.

Example 3.2

Continuous PSC beams, having two equal spans and supporting a floor, are spaced at 5m. Live load on the floor is 400kg/m^2. Total dead load (including self weight of the beam) on any intermediate beam is 1500 kg/m. The beams are designed with concrete cube strength of 350 kg/cm^2 and steel strength of 15000 kg/cm^2. The beam has uniform cross section which is shown in Fig. 3.12. The effective span and area of steel are 10m and 10.89cm^2. The beams are designed assuming that they are cast separately and composite action of slab and beam is not considered.



(b) SECTION AT C

FIG.3.12 TWO SPAN CONTINUOUS BEAM- EXAMPLE 3.2

ALL DIMENSIONS ARE IN mm

One of the intermediate beams is only considered and the reliability analysis of the same at limit state of strength is illustrated.

Live load on any intermediate beam = 2000 kg/m.

There are two possible loading conditions for the live load

(i) Live load on both spans

(ii) Live load on one span only

For the case (i) the critical sections have been found to be B_1 (4.2 m from the end A), C and D_1(4.2 m from the end E) of Fig. 3.12 and for the case (ii) B_2 (5 m from the end A), C and D_2(5 m from the end E).

It is seen from Chapter 2 that strengths of M350 concrete and 7mm ϕ high tensile steel are distributed as N(422.8,56) kg/cm^2 and N(15680,488) kg/cm^2 respectively. Using Monte Carlo method, 10000 samples are generated for the resistance of critical sections B_1, B_2 and C of the beam. From the generated data parameters of the ultimate resisting moments of the critical sections for the events U and O are found out as illustrated in Example 3.1. The results are given in table 3.1.

Load condition (i) : Both spans loaded:

Magnitudes of external bending moments (due to live load and dead load) at the critical sections for different loading conditions are given in table 3.2. From the above table,

Table 3.1. Parameters of the ultimate resisting moments of critical sections of the beam of Fig. 3.12 for PV of σ_{cu} and σ_s

Sl. No.	Section	UR - $N(M_{rum}, s_{ru})$			OR - $EX_{III,s}(M_{rom}, s_{ro})$*		
		P(U)	M_{rum} (tm)	s_{ru} (tm)	P(O)	M_{rom} (tm)	s_{ro}
1	B_1	0.8787	71.17	2.2	0.1213	66.95	21.8
2	B_2	0.8718	69.48	2.13	0.1282	65.40	22.2
3	C	0.9022	75.66	2.36	0.0978	71.06	22.2

*$EX_{III,s}(M_{rom}, s_{ro})$ denotes Type III extremal (smallest) distribution with parameters M_{rom} and s_{ro}.

Table 3.2. Magnitudes of external bending moments at critical sections of the beam of Fig. 3.12 for deterministic load

Sl. No.	Live Loading on Spans	Total External B.M. (tm) at Section		
		B_1^* or B_2	C	D_1^* or D_2
1*	AC & CE	24.26	43.75	24.26
2	AC	31.25	28.13	3.13
3	CE	3.13	28.13	31.25

*Sections B_1 and D_1 correspond to loading case 1.

$$M_{eB_1} = 24.26 \text{ tm} \; ; \; M_{eC} = 43.75 \text{ tm}$$

where M_{eB_1} and M_{eC} are external moments at sections B_1 and C respectively.

Probability of failure of section B_1(or D_1)

Using Eq. 3.35, the conditional probability of failure of the section B_1 for the given event U is

$$p_{f|u} = \phi\left(\frac{24.26 - 71.17}{2.2}\right) = 0$$

Using Eq. 3.36, the conditional probability of failure of the section B_1 for the given event O is

$$p_{f|o} = 1 - \exp\left[-\left(\frac{24.26}{66.95}\right)^{21.8}\right] = 2.44 \times 10^{-10}$$

The probability of failure of the section B_1, p_{fB_1}, using Eq. 3.31, is given by

$$p_{fB_1} = (0.8787)(0) + (0.1213)(2.44 \times 10^{-10})$$

$$= 2.96 \times 10^{-11}$$

Probability of failure of section C

Using Eq. 3.35, the conditional probability of failure of the section C for the given event U is

$$p_{f|u} = \phi\left(\frac{43.75 - 75.66}{2.36}\right) = 0$$

Using Eq. 3.36, the conditional probability of failure of the section C for given event O is

$$p_{f|o} = 1 - \exp\left[-\left(\frac{43.75}{71.06}\right)^{22.2}\right] = 2.1 \times 10^{-5}$$

Using Eq. 3.31, the probability of failure of the section C is

$$p_{fC} = (0.9022)(0) + (0.0978)(2.1 \times 10^{-5})$$
$$= 2.05 \times 10^{-6}$$

Sections B_1 and C should fail simultaneously for the occurrence of failure mode in span AC. The probability of occurrence of the failure mode in span AC, p_{fAC}, using Eq. 3.39, is

$$p_{fAC} = p_{fB_1}\, p_{fC} = 6.1 \times 10^{-17} = p_{fCE}$$

The maximum value of the occurrence of failure mode for this load condition 1 is p_{fAC} or p_{fCE} which is equal to 6.1×10^{-17}. Defining p_{fs_1} as the probability of failure of the beam for loading condition (i), bounds on the value of p_{fs_1}, using Eq. 3.40, are

$$6.1 \times 10^{-17} \leq p_{fs_1} \leq 6.1 \times 10^{-17} + 6.1 \times 10^{-17}$$
$$6.1 \times 10^{-17} \leq p_{fs_1} \leq 12.2 \times 10^{-17}$$

Load Condition (ii): Span AC loaded

From table 3.2,

$$M_{eB_2} = 31.25 \text{ tm} ;\ M_{eC} = 28.13 \text{ tm} ;\ M_{eD} = 3.13 \text{ tm}.$$

Probability of failure of section B_2

Using Eqs. 3.35 and 3.36, the values of conditional probability of failure of the section are

$$p_{f|u} = 0 \ ;\ p_{f|o} = 1.04 \times 10^{-8}$$

The probability of failure of the section B_2 is

$$p_{fB_2} = (0.8718)\,(0) + (0.1282)\,(1.04 \times 10^{-8})$$
$$= 1.33 \times 10^{-9}$$

Probability of failure of section C

Using Eqs. 3.35 and 3.36, the values of conditional probability of failure of the section are

$$p_{f|u} = 0 \quad ; \; p_{f|o} = 1.2 \times 10^{-8}$$

The probability of failure of the section C is

$$p_{fC} = (0.9022)\,(0) + (0.0978)\,(1.2 \times 10^{-8})$$
$$= 1.19 \times 10^{-9}$$

Probability of failure of section D_2

Using Eqs. 3.35 and 3.36, the values of conditional probability of failure of the section are

$$p_{f|u} = 0 \quad ; \; p_{f|o} = 0$$

The probability of failure of the section D_2 is

$$p_{fD_2} = (0.9022)\,(0) + (0.0978)\,(0) = 0$$

The probability of occurrence of the failure mode in span AC, p_{fAC}, is

$$p_{fAC} = p_{fB_2}\, p_{fC}$$
$$= (1.33 \times 10^{-9})\,(1.19 \times 10^{-9}) = 1.58 \times 10^{-18}$$

Similarly, the probability of occurrence of the failure mode in span CE is evaluated and found to be equal to zero(i.e. less than 10^{-24}).

It is found that the value of p_{fCE} is negligible compared to p_{fAC} and so the upper bound and lower bound on the value of p_{fs_2} are same and equal to p_{fAC}. Hence

$$p_{fs_2} = 1.58 \times 10^{-18}$$

Similarly, the probability of failure of the beam, when the live load occupies the span CE only is calculated and given in table 3.3. The bounds on the probability of failure of the beam, p_{fs}, for all loading conditions and failure modes are calculated by using Eq. 3.41 and are given by

$$6.10 \times 10^{-17} \leq p_{fs} \leq 12.52 \times 10^{-17}$$

Example 3.3

Continuous prestressed concrete beams, having three equal spans and supporting a floor, are spaced at 4m. Live load on the floor is 400 kg/m^2. Total dead load (including self weight of the beam) on any intermediate beam is 1000 kg/m. The beams are designed with concrete cube strength of 350 kg/cm^2 and steel strength of 15000 kg/cm^2. The beams have uniform cross section as shown in Fig. 3.13. The effective span and area of steel are 8m and 6.94 cm^2 respectively. The beams are designed assuming they are cast separately and the composite action of slab and beam are not considered.

Table 3.3. Bounds on the PF of the beam of Fig. 3.12 for deterministic load and PV of σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Probability of Failure of Section			Probability of Occurrence of Failure Mode in Span		Bounds on the Probability of Failure of the Beam for each Loading Case
		B_1^+ or B_2	C	D_1^+ or D_2	AC	CE	
1+	AC & CE	2.96(-11)*	2.05(-6)	2.96(-11)	6.10(-17)	6.10(-17)	$6.10(-17) \leq P_{fs_1} \leq 12.2(-17)$
2	AC	1.33(-9)	1.19(-9)	0	1.58(-18)	0	1.58(-18)
3	CE	0	1.19(-9)	1.33(-9)	0	1.58(-18)	1.58(-18)

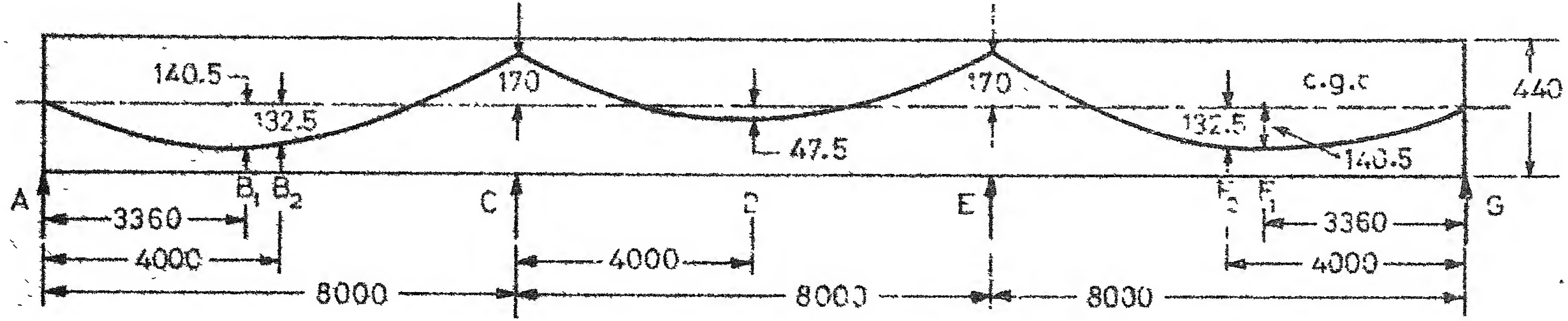
Bounds on the probability of failure of the beam : $6.1\ (-17) \leq P_{fs} \leq 12.57\ (-17)$

+ Sections B_1 and D_1 correspond to loading case 1.

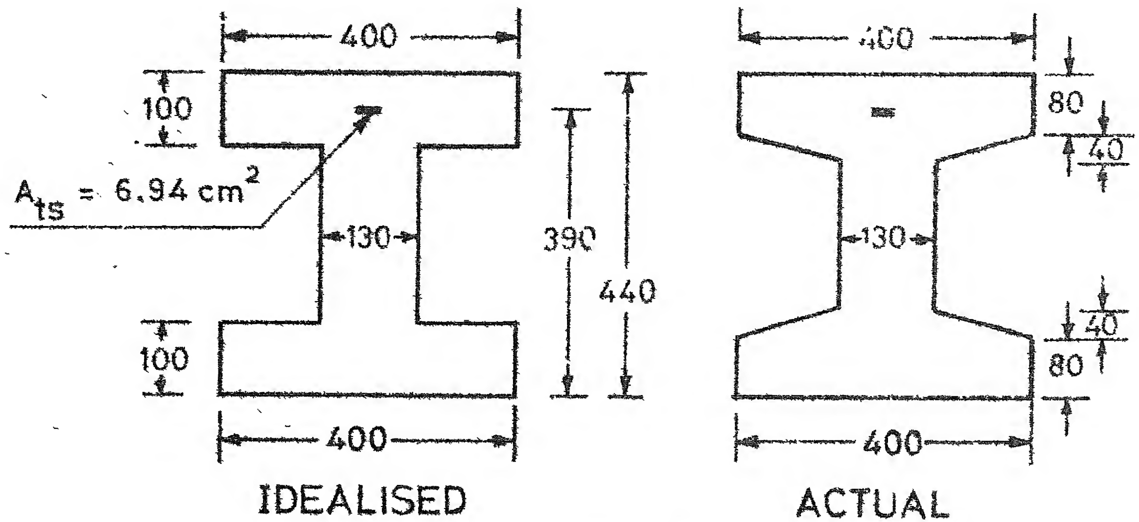
* 2.96(-11) is read as 2.96×10^{-11}

Note : PF denotes probability of failure

PV denotes probabilistic variations.



(a) PARABOLIC CABLE PROFILE



(b) SECTION AT C

FIG.3.13 THREE SPAN CONTINUOUS BEAM-EXAMPLE 3.3

ALL DIMENSIONS ARE IN mm

One of the intermediate beams only is considered and the reliability analysis of the same at limit state of strength is illustrated.

Live load on any intermediate beam = 4 × 400 = 1600 kg/m.

There are five possible loading conditions for the live load acting on the beam as stated below:

(i) Live load on all spans

(ii) Live load on end spans AC and EG

(iii) Live load on middle span CE

(iv) Live load on end span AC(or EG) only and

(v) Live load on two adjacent spans.

The critical sections are found to be B_1(3.36m from the end A), C, D, E and F_1 (3.36m from the end G) of Fig. 3.13 for cases (i) and (v) and B_2 (4m from the end A), C, D, E and F_2 (4m from the end G) for cases (ii), (iii) and (iv).

It is known from Chapter 2 that strengths of M350 concrete and 7mm ϕ high tensile steel are distributed as N(422.8, 56) kg/cm^2 and N(15680, 488) kg/cm^2 respectively. Using Monte Carlo technique, parameters of the ultimate resisting moments of critical sections for events U and O are obtained and given in table 3.4.

Magnitudes of the external bending moments (due to dead load and live load) at the critical sections for different loading cases are shown in table 3.5. Failure probabilities of the beam are calculated for different loading cases.

Table 3.4. Parameters of the ultimate resisting moments of critical sections of the beam of Fig. 3.13 for PV of σ_{cu} and σ_s

Sl. No.	Section	UR - N(M_{rum}, s_{ru})			OR - $EX_{III,s}$ (M_{rom}, s_{ro})		
		P(U)	M_{rum} (tm)	s_{ru} (tm)	P(O)	M_{rom} (tm)	s_{ro}
1	B_1	0.8329	34.41	1.06	0.1671	32.35	21.3
2	B_2	0.8175	33.50	1.04	0.1825	31.55	21.0
3	C	0.8757	37.60	1.16	0.1243	35.33	22.2
4	D	0.6072	24.30	0.75	0.3928	22.78	18.7

Table 3.5. Magnitudes of external bending moments at critical sections of the beam of Fig. 3.13 for deterministic load

Sl. No.	Live Load on Spans	Total External B.M. (tm) at Section				
		B_1^* or B_2	C	D	E	F_1^* or F_2
1*	AC,CE & EG	13.28	16.64	4.16	16.64	13.28
2	AC & EG	15.04	11.52	3.52	11.52	15.04
3	CE	2.24	11.52	9.27	11.52	2.24
4	AC	14.16	13.26	0.116	4.66	5.67
5*	AC & CE	12.55	18.38	6.72	9.78	1.03
6	EG	5.67	4.66	0.116	13.26	14.16
7*	CE & EG	1.03	9.78	6.72	18.38	12.55

*Sections B_1 and F_1 correspond to loading cases 1, 5 and 7.

Load Condition (i): Live load on all spans

From table 3.5, the values of bending moments at the critical sections are

$$M_{eB_1} = M_{eD_1} = 13.28 \text{ tm} ;$$

$$M_{eC} = M_{eE} = 16.64 \text{ tm} ; \; M_{eD} = 4.16 \text{ tm}.$$

The conditional probabilities of failure of the critical sections B_1, C, D, E and F_1 for events U and O are calculated as illustrated in the previous examples and are given in table 3.6. The probability of failure of the above sections, calculated by using Eq. 3.31 are

$$p_{fB_1} = p_{fD_1} = 9.7 \times 10^{-10}$$

$$p_{fC} = 6.8 \times 10^{-9} \; ; \; p_{fD} = 6.1 \times 10^{-15}$$

There are 3 failure modes in three span continuous beams. Sections B_1 and C should fail for the occurrence of failure mode in the span AC. Its probability is given by

$$p_{fAC} = p_{fB_1} \, p_{fC} = 6.6 \times 10^{-18} = p_{fEG}$$

Sections C,D and E should fail for the occurrence of failure mode in the span CE. Its probability of failure is

$$p_{fCE} = p_{fC} \cdot p_{fD} \cdot p_{fE}$$

$$= 2.8 \times 10^{-31}$$

Bounds on the value of p_{fs_1}, using Eq. 3.34, are

$$6.6 \times 10^{-18} \leq p_{fs_1} \leq 13.2 \times 10^{-18}$$

Table 3.6. PF of critical sections of Fig. 3.13 for deterministic load and PV of σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Probability of Failure of Sections									
		B_1^+ or B_2		C		D		E		F_1^+ or F_2	
		UR	OR	UR	OR	UR	OR	UR	OR	UR	OR
1+	AC,CE & EG	0	9.7(-10)*	0	6.8(-9)	0	6.1(-15)	0	6.8(-9)	0	9.7(-10)
2	AC & EG	0	3.2(-8)	0	2 (-12)	0	2.6(-16)	0	2.0(-12)	0	3.2(-8)
3	CE	0	0	0	2 (-11)	0	2.0(-8)	0	2.0(-11)	0	0
4	AC	0	9.0(-9)	0	4.4(-11)	0	0	0	0	0	4.0(-18)
5+	AC & CE	0	2.9(-10)	0	6.2(-8)	0	4.8(-11)	0	5.1(-14)	0	0.0
6	EG	0	4.0(-18)	0	0	0	0	0	4.4(-11)	0	9.0(-9)
7+	CE & EG	0	0	0	5.1(-14)	0	4.8(-11)	0	6.2(-8)	0	2.9(-10)

+ Sections B_1 and F_1 correspond to loading cases 1,5 and 7.

* 9.7(-10) is read as 9.7×10^{-10}.

Similarly the reliability analysis is done for other load conditions and the bounds on the probability of failure of the beam for different load conditions are given in table 3.7.

From the above table, the bounds on the p_{fs} for all loading conditions and failure modes are equal to:

$$1.8 \times 10^{-17} \leq p_{fs} \leq 5.01 \times 10^{-17}$$

3.10 PROBABILITY DISTRIBUTION OF M_r OF A SECTION FOR PROBABILISTIC VARIATIONS OF σ_{cu}, σ_s AND DIMENSIONS OF SECTION

In previous article, the probabilistic variations of strengths of materials only were considered in the resistance of the section; but in practice, as observed in Chapter 2, there are random discrepancies between actual and nominal cross-sectional dimensions of structural members, areas of steel bars and position of reinforcement. The differences between actual and nominal dimensions, expressed as percentage of the nominal dimensions, are generally not independent of these dimensions. Since lack of data on the variations of these differences with nominal dimensions for Indian conditions, the ratios of the mean dimensions to their respective characteristic values and coefficients of variation of parameters established for the section of Fig. 2.19 in Chapter 2, are assumed same for all sections in this thesis.

Random variations of the variables b_t, b', t_t, d and D_s in the equations given in the article 3.5, are considered in the

Table 3.7. Bounds on the PF of the beam of Fig. 3.13 for deterministic load and PV of σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Probability of Failure of Sections					Probability of Occurrence of Failure Mode in the Span			Bounds on the PF of the Beam for each Loading Case
		B_1^+ or B_2	C	D	E	F_1^+ or F_2	AC	CE	EG	
1+	AC, CE & EG	9.7(-10)*	6.8(-9)	6.1(-15)	6.8(-9)	9.7(-10)	6.6(-18)	2.8(-31)	6.6(-18)	$6.6(-18) \leq p_{fs_1} \leq 13.2(-18)$
2	AC & EG	3.2(-8)	2.0(-12)	2.6(-16)	2.0(-12)	3.2(-8)	6.4(-20)	1.0(-39)	6.4(-20)	$6.4(-20) \leq p_{fs_2} \leq 12.8(-20)$
3	CE	0	2.0(-11)	2.0(-8)	2.0(-11)	0	0	8.0(-30)	0	8.00(-30)
4	AC	9.0(-9)	4.4(-11)	0	0	4.0(-17)	4.0(-19)	0	0	4.00(-19)
5+	AC & CE	2.9(-10)	6.2(-8)	4.8(-11)	5.1(-14)	0	1.8(-17)	1.5(-31)	0	1.80(-17)
6	EG	4.0(-17)	0	0	4.4(-11)	9.0(-9)	0	0	4.0(-19)	4.00(-19)
7+	CE & EG	0	5.1(-14)	4.8(-11)	6.2(-8)	2.9(-10)	0	1.5(-31)	1.8(-17)	1.80(-17)

Bounds on probability of failure of the beam : $1.80(-17) \leq p_{fs} \leq 5.01(-17)$

+ Sections B_1 and F_1 correspond to loading cases 1, 5 and 7.

* 9.7(-10) is read as 9.7×10^{-10}.

resistance of the section. Using the ratios of the mean values to their respective characteristic values of b_t, b', t_t and d and their coefficients of variation, the mean values and standard deviations of b_t, b', t_t and d of the section shown in Fig. 3.2 can be calculated. These values are given in table 3.8. Using normal distribution (observed in Chapter 2) for all random variables b_t, b', t_t, d, D_s, σ_{ou} and σ_s with their parameters, Monte Carlo simulation method is used and the distribution of the resistance of the section is obtained. The histogram and cumulative distribution of M_r of the section for 10000 samples are given in Fig. 3.14. Normal, beta, Type I extremal and Type III extremal distributions have been tried for the generated data and none of them is found to satisfy the chi square test. The samples for over-reinforced and under-reinforced cases are arranged separately and their histograms and cumulative distributions are shown in Figs. 3.15 and 3.16. It is observed that about 4.4 percent of the samples lie in the over-reinforced case even though the beam is under-reinforced deterministically. It is found that the normal and Type III extremal (smallest) distributions satisfy the chi square test of the generated data of Figs. 3.15 and 3.16 respectively for one percent level of significance. Hence the normal distribution for under-reinforced case and Type III extremal (smallest) distribution for over-reinforced case are adopted in the reliability analysis of the PSC beams at limit state of strength. The formulation of the

Table 3.8. Mean values and standard deviations of dimensions of the section shown in Fig. 3.2.

Sl. No.	Item	Mean Value / Characteristic Value	Coefft. of Variation	Characteristic value (cm)	Mean Value (cm)	Standard Deviation (cm)	Distribution
1	b_t	1.031	0.00794	60	61.86	0.491	Normal
2	b'	0.995	0.00918	10	9.95	0.091	Normal
3	t_t	1.072	0.0507	10	10.72	0.544	Normal
4	d	1.083	0.0384	72	77.976	2.994	Normal

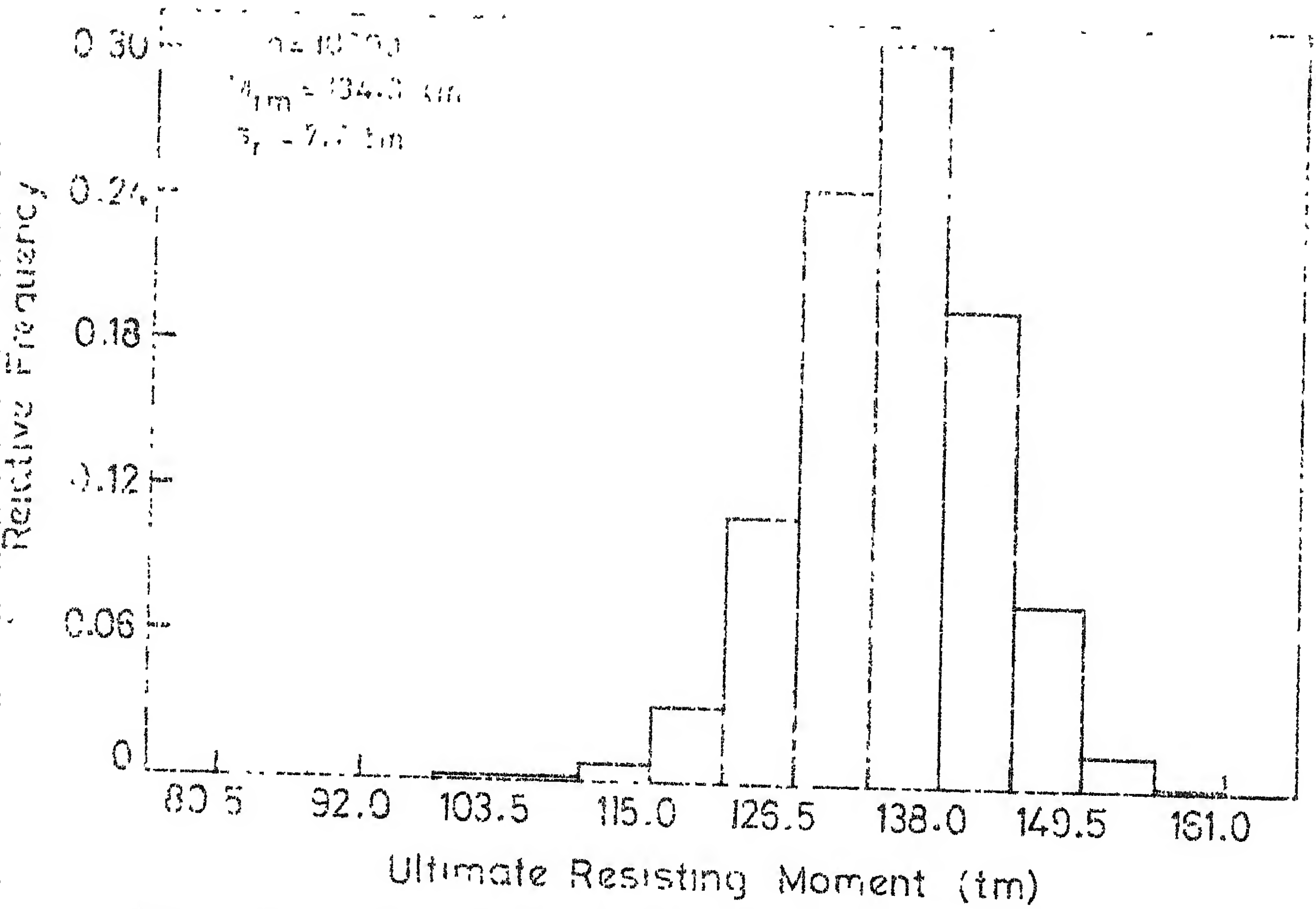
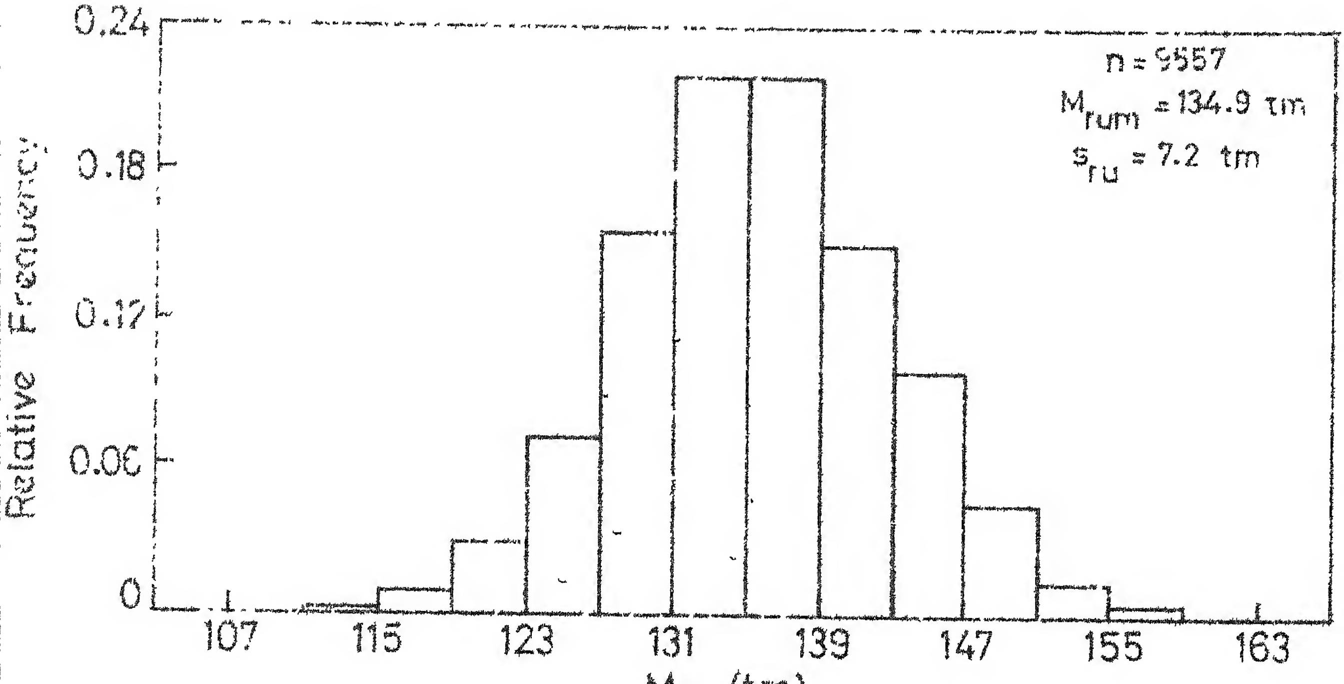
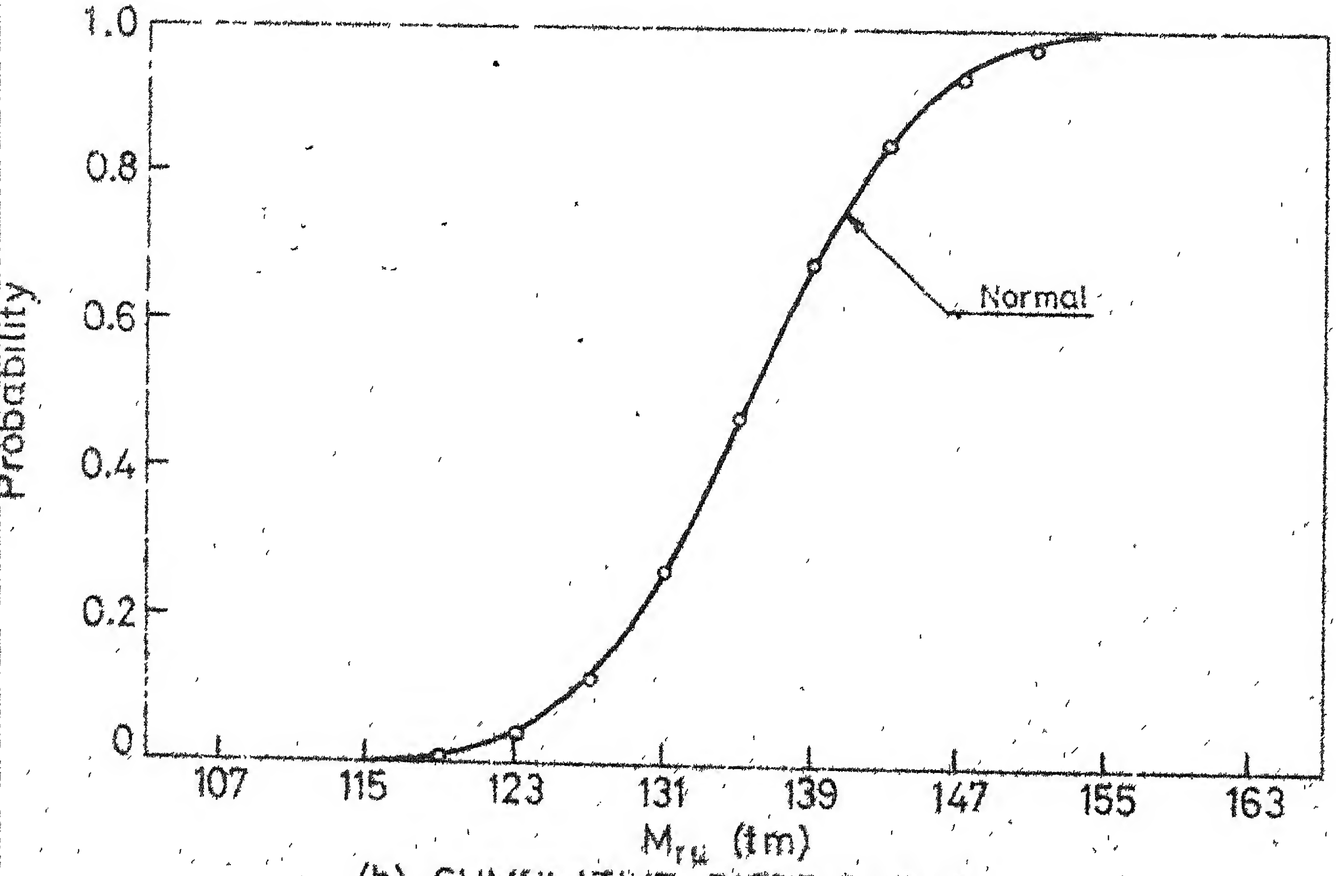


FIG.3.14 HISTOGRAM OF M_r OF SECTION OF FIG. 3.2 FOR PV OF σ_{cu}, σ_s AND DIMENSIONS OF SECTION

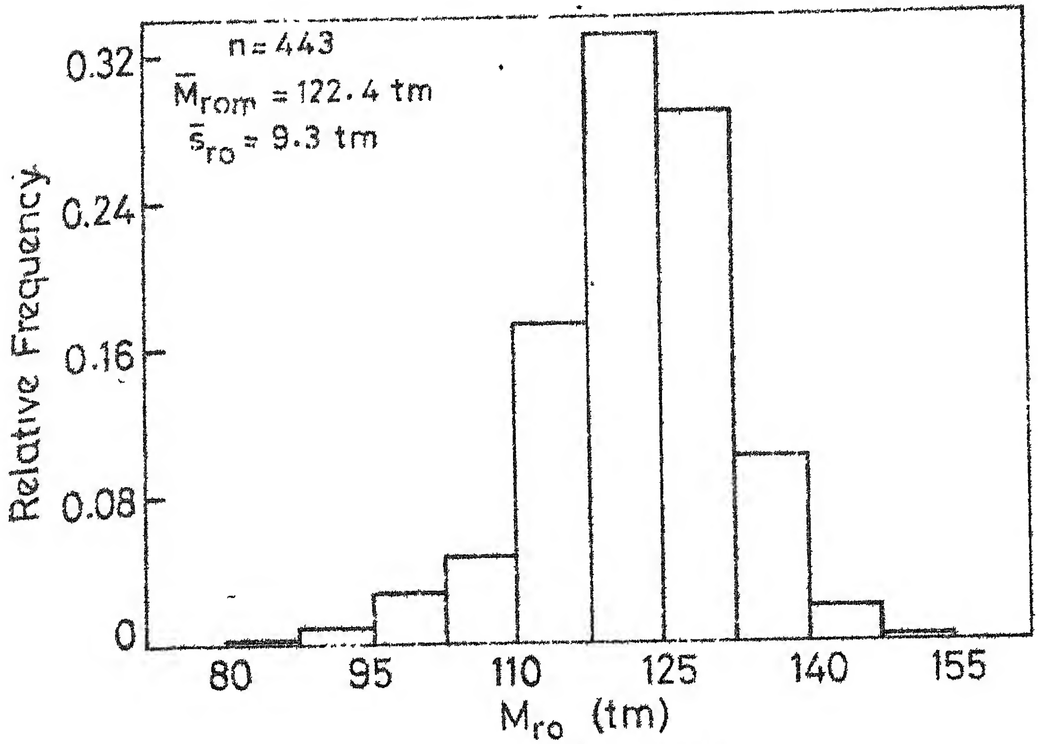


(a) HISTOGRAM

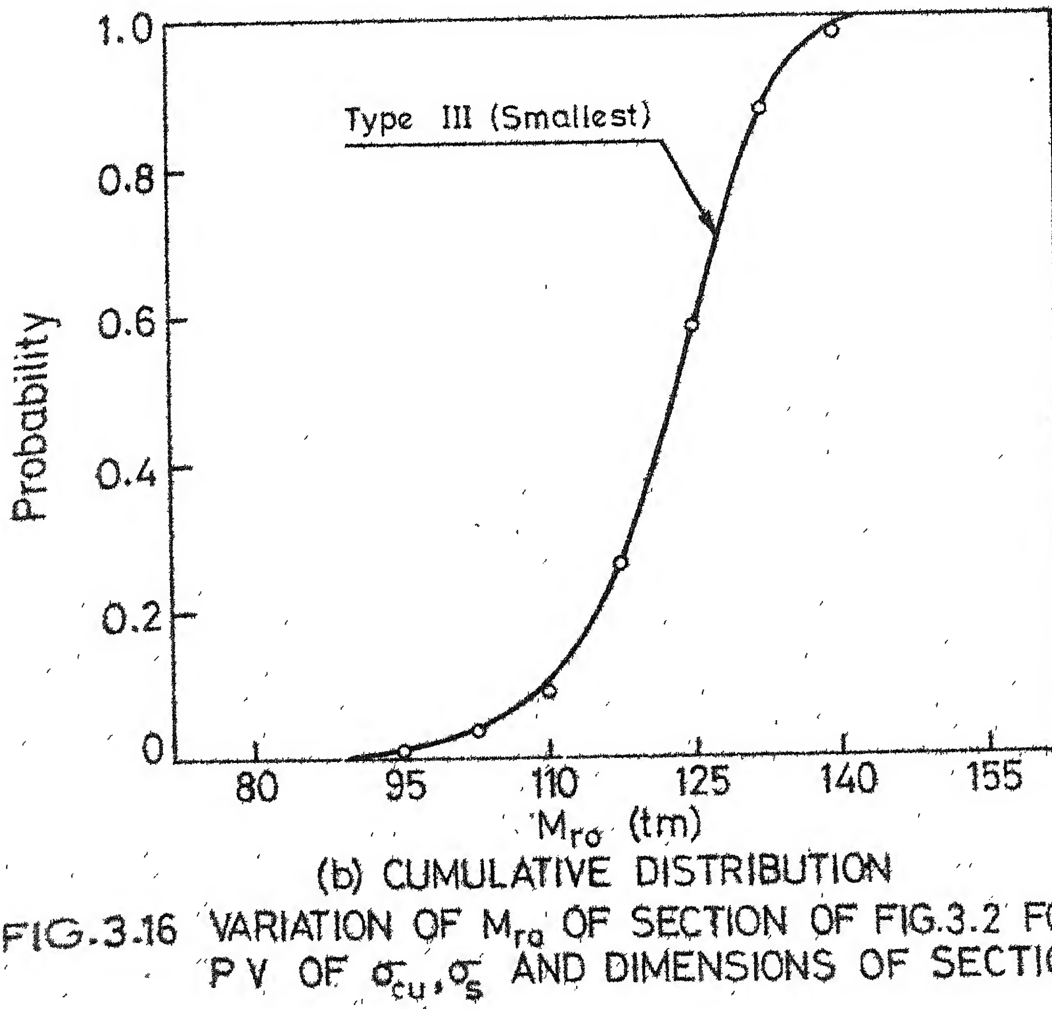


(b) CUMULATIVE DISTRIBUTION

FIG. 3.15 VARIATION OF M_{ru} OF SECTION OF FIG. 3.2 FOR P V OF σ_{cu}, σ_{c} AND DIMENSIONS OF SECTION



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG.3.16 VARIATION OF M_{ro} OF SECTION OF FIG.3.2 FOR P V OF σ_{cu}, σ_s AND DIMENSIONS OF SECTION

the reliability analysis at limit state of strength is same as explained in article 3.7. The procedure is illustrated with examples.

Example 3.4

The simply supported PSC beam of Example 3.1 (Fig. 3.11) is considered for probabilistic variations of strengths of materials and geometric properties of the section. Other particulars of the beam remain same as in Example 3.1.

Using the coefficients of variation and the ratios of the mean values of the parameters b_t b', t_t, h and d to their respective characteristic values (found in Chapter 2), the mean values and standard deviations of b_t, b', t_t, h and d of the beam shown in Fig. 3.11 are calculated as follows:

$$b_{tm} = 1.031 \times 50 = 51.55\,cm$$
$$b'_m = 0.995 \times 15 = 14.925\ cm$$
$$t_{tm} = 1.072 \times 12 = 13.936\,cm$$
$$d_m = 1.083 \times 54.7 = 59.24\,cm$$
$$h_m = 1.031 \times 61 = 62.89\ cm$$
$$s_{bt} = 0.00794 \times b_{tm} = 0.409\,cm$$
$$s_{b'} = 0.00918 \times b'_m = 0.137\,cm$$
$$s_{tt} = 0.0507\ t_{tm} = 0.7065\,cm$$
$$s_d = 0.0384\ d_m = 2.079\,cm$$
$$s_h = 0.0065\ h_m = 0.409\,cm$$

It is known from Chapter 2,

$$\sigma_{cum} = 422.8 \text{ kg/cm}^2 \; ; \; s_c = 56 \text{ kg/cm}^2$$
$$\sigma_s = 15680 \text{ kg/cm}^2 \; ; \; s_s = 488 \text{ kg/cm}^2$$
$$D_{sm} = 7.065 \text{ mm} \; ; \; s_D = 0.0345 \text{ mm}$$

Using normal distribution for all the above random variables, 20,000 samples are generated for M_r applying Monte Carlo method. The number of samples in events U and O are equal to 18600 and 1400 respectively. Using Eq. 3.32,

$$P(O) = 0.07 \quad \text{and} \quad P(U) = 0.93$$

For the under-reinforced case (normal)

$$M_{rum} = 100.51 \text{ tm} \; ; \; s_{ru} = 5.37 \text{ tm}$$

For the over-reinforced case (Type III extremal smallest)

$$M_{rom} = 94.67 \text{ tm} \; ; \; s_{ro} = 17.0$$

Using Eqs. 3.35 and 3.36, $p_{f|u}$ and $p_{f|o}$ are found to be equal to zero and 2.0×10^{-6} respectively. The probability of failure of the beam is

$$p_f = (0.93)\,(0) + (0.07)\,(2.0 \times 10^{-6})$$
$$= 1.4 \times 10^{-7}$$

The method of reliability analysis of continuous PSC beams is same as explained in article 3.8 except that the parameters of the resistance for events U and O are to be obtained taking probabilistic variations of strengths of materials and geometric properties of the

section as illustrated in the previous example. The results of the reliability analysis of the two span beam of Fig. 3.12 are given in tables 3.9 to 3.12 and for the three span beam of Fig. 3.13 in tables 3.13 to 3.16.

3.11 COMPUTATION OF PROBABILITY OF FAILURE FOR PROBABILISTIC VARIATION OF LOAD

For probabilistic variation of action (load) and resistance, the probability of failure is given by

$$p_f = P(R < S) = \int_{-\infty}^{\infty} f_R(r) \left[\int_{r}^{\infty} f_S(s)\, ds \right] dr \qquad (3.42)$$

where $f_R(r)$ and $f_S(s)$ are probability density functions of R and S respectively.

The above Eq. 3.42 can be rewritten as

$$p_f = P(R < S) = \int_{-\infty}^{\infty} f_R(r) \left[1 - F_S(r) \right] dr \qquad (3.43)$$

where $F_S(r)$ is the cumulative distribution of S. The area of the hatched portion in the Fig. 3.17 gives the probability of failure.

(a) Lognormal Distribution for Load and Normal Distribution for Resistance

The probability distribution for the live load has been found to be lognormal in Chapter 2. Since the bending moment is directly proportional to load, the probability distribution of M_e is also lognormal which is given by

Table 3.9. Mean values and standard deviations of dimensions of the sections of the beam of Fig. 3.12

Sl. No.	Item	Mean Value / Characteristic Value	Coefft. of Variation	Characteristic Value (cm)	Mean Value (cm)	Standard Deviation (cm)	Distribution
1	b_t	1.031	0.00794	50	51.55	0.409	Normal
2	b'	0.995	0.00918	15	14.93	0.137	Normal
3	t_t	1.072	0.0507	12	13.94	0.706	Normal
4	d_{B_1}	1.083	0.0384	47.36	51.29	1.970	Normal
5	d_{B_2}	1.083	0.0384	46.35	50.20	1.928	Normal
6	d_C	1.083	0.0384	50	54.15	2.019	Normal
7	h	1.031	0.0065	56	57.74	0.375	Normal

Table 3.10. Parameters of the ultimate resisting moments of critical sections of the beam of Fig. 3.12 for PV of σ_{cu}, σ_s and dimensions of section

Sl. No.	Section	UR - $N(M_{rum}, s_{ru})$			OR - $EX_{III,s}(M_{rom}, s_{ro})$		
		P(U)	M_{rum} (tm)	s_{ru} (tm)	P(O)	M_{rom} (tm)	s_{ro}
1	B_1	0.9512	79.14	4.25	0.0488	74.70	17.0
2	B_2	0.9476	77.18	4.23	0.0524	73.12	16.0
3	C	0.9613	84.17	4.59	0.0387	79.06	17.7

Table 3.11. PF of critical sections of the beam of Fig. 3.12 for deterministic load and PV of σ_{cu} , σ_s and dimensions of section

Sl. No.	Live Loading of Spans	Probability of Failure of Sections					
		B_1^+ or B_2		C		D_1^+ or D_2	
		UR	OR	UR	OR	UR	OR
1+	AC & CE	0	2.41(-10)	0	1.10(-6)	0	2.17(-10)
2	AC	0	1.50(-8)	0	2.84(-9)	0	0
3	CE	0	0	0	2.84(-9)	0	1.50(-8)

Table 3.12. Bounds on the PF of the beam of Fig. 3.12 for deterministic load and PV of σ_{cu} , σ_s and dimensions of section

Sl. No.	Live Loading of Spans	Probability of Failure of Section			Probability of Occurrence of Failure Mode in the Span		Bounds on PF of the Beam for each Loading Case
		B_1^+ or B_2	C	D_1^+ or D_2	AC	CE	
1+	AC & CE	2.41(-10)*	1.10(-6)	2.41(-10)	2.65(-16)	2.65(-16)	2.65(-16) $\leq p_{fs_1} \leq$ 5.30(-16)
2	AC	1.50(-8)	2.84(-9)	0	4.26(-17)	0	4.26(-17)
3	CE	0	2.84(-9)	1,50(-8)	0	4.26(-17)	4.26(-17)

Bounds on the probability of failure of the beam : 2.65(-16) $\leq p_{fs} \leq$ 6.16(-16)

+ Sections B_1 and D_1 correspond to loading case 1.

* 2.41(-10) is read as 2.41×10^{-10}.

Table 3.13. Mean values and standard deviations of dimensions of the sections of the beam of Fig. 3.13.

Sl. No.	Item	Mean Value / Characteristic value	Coefft. of Variation	Characteristic Value (cm)	Mean Value (cm)	Standard Deviation (cm)	Distribution
1	b_t	1.031	0.00794	40.0	41.24	0.327	Normal
2	b'	0.995	0.00918	13.0	12.94	0.119	Normal
3	t	1.072	0.0507	10.0	10.72	0.544	Normal
4	d_{B_1}	1.083	0.0384	36.05	39.04	1.499	Normal
5	d_{B_2}	1.083	0.0384	35.25	38.18	1.466	Normal
6	d_C	1.083	0.0384	39.0	42.24	1.622	Normal
7	d_D	1.083	0.0384	26.75	28.97	1.112	Normal
8	h	1.031	0.0065	44.0	45.36	0.295	Normal

Table 3.14. Parameters of the ultimate resisting moments of critical sections of the beam of Fig. 3.13 for PV of σ_{cu}, σ_s and dimensions of section

Sl. No.	Section	UR - N(M_{rum}, s_{ru})			OR - $EX_{III,s}$(M_{rom}, s_{ro})		
		P(U)	M_{rum} (tm)	s_{ru} (tm)	P(O)	M_{rom} (tm)	s_{ro}
1	B_1	0.9285	38.26	2.1	0.0715	36.10	16.9
2	B_2	0.9213	37.28	2.01	0.0787	35.18	17.0
3	C	0.9503	41.80	2.28	0.0497	39.34	17.4
4	D	0.7538	27.10	1.49	0.2462	25.60	16.0

Table 3.15. PF of critical sections of the beam of Fig. 3.13 for deterministic load and PV of σ_{cu} , σ_s and dimensions of section

Sl. No.	Live Load on Spans	Probability of Failure of Sections									
		B_1^+ or B_2		C		D		E		F_1^+ or F_2	
		UR	OR	UR	OR	UR	OR	UR	OR	UR	OR
1+	AC, CE & EG	0	3.3(-9)*	0	1.6(-8)	0	5.8(-14)	0	1.6(-8)	0	3.3(-9)
2	AC & EG	0	4.2(-8)	0	2.6(-11)	0	4.0(-15)	0	2.6(-11)	0	4.2(-8)
3	CE	0	0	0	2.6(-11)	0	2.2(-9)	0	2.6(-11)	0	0
4	AC	0	1.5(-8)	0	3.0(-10)	0	0	0	0	0	2.6(-15)
5+	AC & CE	0	1.3(-9)	0	8.8(-8)	0	1.3(-10)	0	7.4(-12)	0	0
6	EG	0	2.6(-15)	0	0	0	0	0	3.0(-10)	0	1.5(-8)
7+	CE & EG	0	0	0	7.4(-12)	0	1.3(-10)	0	8.8(-8)	0	1.3(-9)

+ Sections B_1 and F_1 correspond to loading cases 1,5 and 7.

* 3.3(-9) is read as 3.3×10^{-9}.

Table 3.16. Bounds on the PF of the beam of Fig. 3.13 for deterministic load and PV of σ_{cu}, σ_s and dimensions of section

Sl. No.	Live Load on Spans	Probability of Failure of Sections					Probability of Occurrence of Failure Mode in Span			Bounds on the PF of the Beam for each Loading Case
		B_1^+ or B_2	C	D	E	F_1^+ or E_2	AC	CE	EG	
1+	AC, CE & EG	3.3(-9)*	1.6(-8)	5.8(-14)	1.6(-8)	3.3(-9)	5.3(-17)	1.5(-29)	5.3(-17)	5.3(-17) $\leq p_{fs_1} \leq$ 10.6(-17)
2	AC & EG	4.2(-8)	2.6(-11)	4.0(-15)	2.6(-11)	4.2(-8)	1.2(-18)	2.7(-36)	1.2(-18)	1.2(-18) $\leq p_{fs_2} \leq$ 2.4(-18)
3	CE	0	2.6(-11)	2.2(-9)	2.6(-11)	0	0	1.5(-30)	0	1.5(-30)
4	AC	1.5(-8)	3.0(-10)	0	0	2.6(-15)	4.5(-18)	0	0	4.5(-18)
5+	AC & CE	1.3(-9)	8.8(-8)	1.3(-10)	7.4(-12)	0	1.1(-16)	8.5(-29)	0	1.1(-16)
6	EG	2.6(-15)	0	0	3.0(-10)	1.5(-8)	0	0	4.5(-18)	4.5(-18)
7+	CE & EG	0	7.4(-12)	1.3(-10)	8.8(-8)	1.3(-9)	0	8.5(-29)	1.1(-16)	1.1(-16)

Bounds on the probability of failure of the beam : 1.1(-16) $\leq p_{fs} \leq$ 2.42 (-16)

+ Sections B_1 and F_1 correspond to loading cases 1, 5 and 7.

* 3.3(-9) is read as 3.3×10^{-9}.

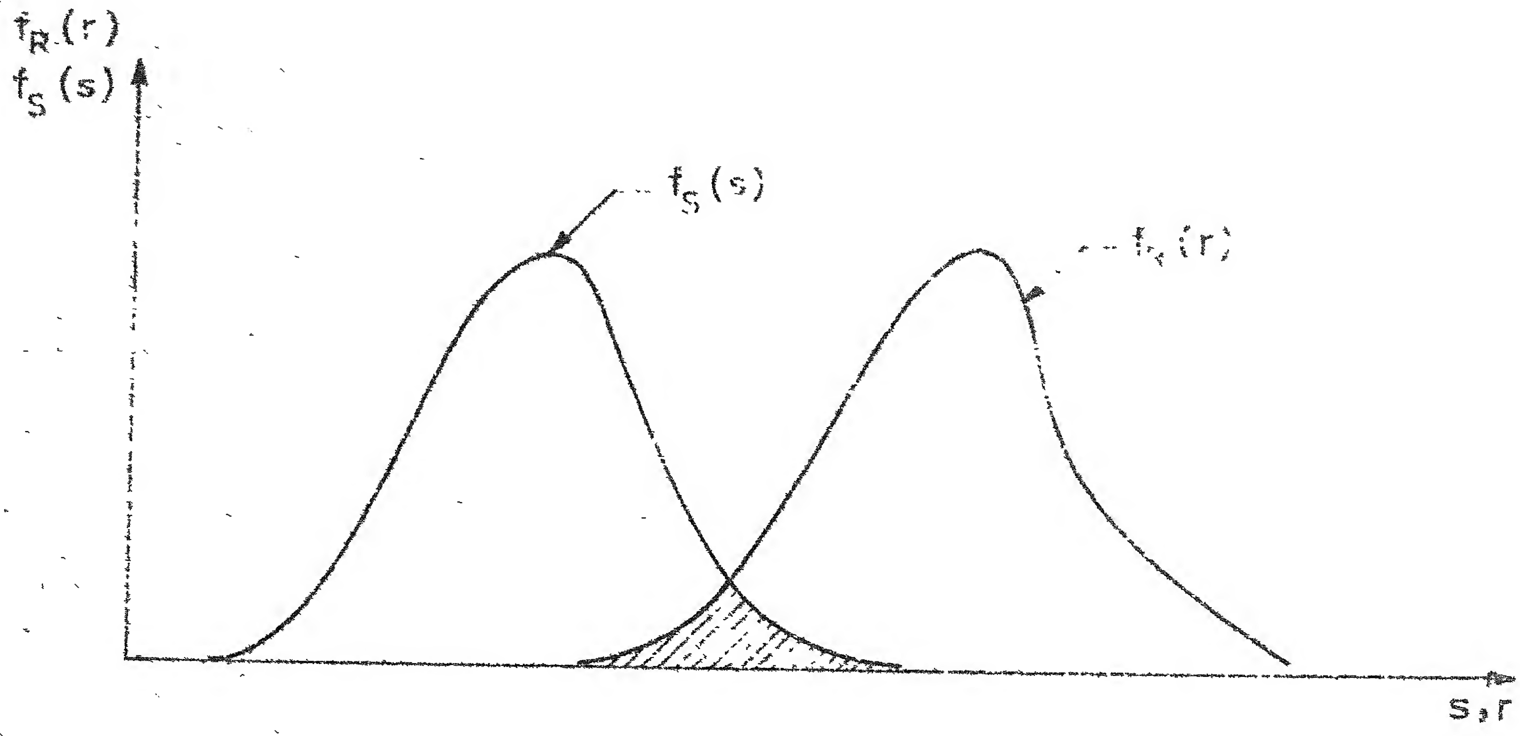


FIG. 3.17 FREQUENCY DISTRIBUTION FUNCTION OF R AND S

$$F_{M_e}(M_e) = \phi\left[\frac{\log\left(\frac{M_e}{M_{em}}\right)}{s_{me}}\right] \tag{3.44}$$

where $F_{M_e}(M_e)$ = cumulative distribution of M_e

M_{em} & s_{me} = parameters of M_e following lognormal distribution.

When the beam fails in tension (under-reinforced), it has been found in the article 3.5 that the probability distribution of the resistance of the section is normal. Hence using Eq. 3.25 for the probability density function of the resistance and substituting Eq. 3.44 in Eq. 3.43, the probability of failure is obtained as

$$p_f = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\, s_{ru}} \exp\left[-\frac{1}{2}\left(\frac{M_{ru} - M_{rum}}{s_{ru}}\right)^2\right] \times$$

$$\left[1 - \phi\left\{\frac{\log(M_{ru}/M_{em})}{s_{me}}\right\}\right] dM_{ru} \tag{3.45}$$

Substituting

$$w = \frac{1}{\sqrt{2}}\left(\frac{M_{ru} - M_{rum}}{s_{ru}}\right) \tag{3.46}$$

the Eq. 3.45 simplifies to

$$p_f = \int_{-\infty}^{\infty} e^{-w^2} \frac{1}{\sqrt{\pi}} \left[1 - \phi\left\{\frac{\log\left(\frac{\sqrt{2}\, w\, s_{ru} + M_{rum}}{M_{em}}\right)}{s_{me}}\right\}\right] dw \tag{3.47}$$

The external bending moment (M_e) consists of live load bending moment (M_q) and dead load bending moment (M_g):

$$M_e = M_g + M_q \qquad (3.48)$$

Assuming M_g as deterministic, the cumulative probability of M_e can be calculated from the known distribution of M_q.

$$P(M_e < M_{ru}) = P(M_g + M_q < M_{ru}) \qquad (3.49a)$$

$$= P(M_q < M_{ru} - M_g) \qquad (3.49b)$$

Hence

$$F_{M_e}(M_{ru}) = \phi \left[\frac{\log\left(\frac{M_{ru} - M_g}{M_{qm}}\right)}{s_{mq}} \right] \qquad (3.50)$$

where M_{qm} and s_{mq} are parameters of live load bending moment (log-normal). As M_q is proportional to Q, s_{mq} is equal to the dimensionaless parameter s_q. Substituting the Eq. 3.50 in Eq. 3.43, the probability of failure is given by

$$p_f = \int_{-\infty}^{\infty} e^{-w^2} \frac{1}{\sqrt{\pi}} \left[1 - \phi \left\{ \frac{\log\left(\frac{\sqrt{2}\, w\, s_{ru} + M_{rum} - M_g}{M_{qm}}\right)}{s_q} \right\}\right] dw \qquad (3.51)$$

(b) Lognormal Distribution for Load and Type III Extremal (smallest) Distribution for Resistance

When the beam fails in compression (over-reinforced), the probability distribution for the resistance of the section has been found to be Type III extremal (smallest) in the article 3.5. Using Eq. 3.27 for the probability density function of the resistance, and following similar procedure of case (a), the probability of failure is obtained as

$$p_f = \int_0^\infty e^{-w} \left[1.0 - \phi \left\{ \frac{\log \left(\frac{M_{rom} w^{\left(\frac{1}{s_{ro}}\right)} - M_g}{M_{qm}} \right)}{s_q} \right\} \right] dw \tag{3.52}$$

Using numerical quadrature method, the integral equations 3.51 and 3.52 can be evaluated. The above equations are in the standard forms and are evaluated in this thesis using Hermite-Gauss quadrature formula and Laguerre-Gauss quadrature formula (57).

The parameters of the resistance of a section for events U and O are obtained using Monte Carlo technique. The conditional probability of failures $p_{f|u}$ and $p_{f|o}$ are evaluated using Eqs. 3.51 and 3.52, for probabilistic variation of live load. Further reliability analysis of simply supported and continuous beams is same as explained in the articles 3.7 and 3.9. The procedure is illustrated with examples.

Example 3.5

The simply supported PSC beam of Example 3.1 (Fig. 3.11) is considered for probabilistic variations of live load and strengths of materials. All particulars of the beam remain same as in Example 3.1.

It is known from Chapter 2 that the live load is distributed as LN(121.77 kg/m^2, 0. 368). The mode value of the live load bending moment is

$$M_{qm} = \frac{121.77 \times 5 \times 10^2}{8} = 7610 \text{ kg m}$$

Since the bending moment is proportional to the load, M_q is distributed as (7.61 tm, 0.368). The following results are taken from Example 3.1.

$$M_g = 18.75 \text{ tm}$$

$$P(U) = 0.837 \text{ tm} \quad ; \quad P(O) = 0.163$$

$$M_{rum} = 90.57 \text{ tm} \quad ; \quad s_{ru} = 2.84 \text{ tm}$$

$$M_{rom} = 84.80 \text{ tm} \quad ; \quad s_{ro} = 21.6$$

Substituting the above values in Eqs. 3.51 and 3.52 and using numerical quadrature method (57), the values of $p_{f|u}$ and $p_{f|o}$ are obtained as 6.86×10^{-10} and 6.8×10^{-9} respectively. Using Eq. 3.31, the p_f of the section is

$$p_f = (0.837)\,(6.86 \times 10^{-10}) + (0.163)\,(6.8 \times 10^{-9})$$
$$= 1.682 \times 10^{-9}$$

The reliability analysis of continuous beams for probabilistic variation of live load is same as explained in article 3.9 except that probabilistic variation of live load is to be considered and conditional probability of failures are evaluated using Eqs. 3.51 and 3.52. The results of the reliability analysis of the two span beam of Fig. 3.12 in tables 3.17 to 3.19 and of the three span beam of Fig. 3.13 in tables 3.20 to 3.22 for probabilistic variations of Q , σ_{cu} and σ_s are presented.

The method of reliability analysis of beams for probabilistic variations of live load, strengths of materials and geometric properties of the section, is illustrated with an example.

Table 3.17. Magnitudes of external bending moments (mode values of M_q) at critical sections of the beam of Fig. 3.12 for probabilistic live load

Sl. No.	External Bending Moment due to	Loading of Spans	External B.M. (tm) at Section				
			B_1	B_2	C	D_1	D_2
1	Dead load	AC & CE	10.395	9.375	18.75	10.395	9.375
2	Live load	AC & CE	4.219	-	7.61	4.219	-
3	"	AC	-	5.85	3.81	-	1.905*
4	"	CE	-	1.905*	3.81	-	5.85

* These values of M_q are acting opposite to M_g.

Table 3.18. PF of critical sections of the beam of Fig. 3.12 for PV of Q, σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Probability of Failure of Section					
		B_1^+ or B_2		C		D_1^+ or D_2	
		U R	O R	U R	O R	U R	O R
1+	AC & CE	2.41(-13)	3.77(-13)	2.57(-8)	2.66(-8)	2.41(-13)	3.77(-13)
2	AC	1.29(-10)	1.42(-10)	1.31(-13)	2.53(-13)	0	0
3	CE	0	0	1.31(-13)	2.53(-13)	1.29(-10)	1.42(-10)

Table 3.19. Bounds on the PF of the beam of Fig. 3.12 for PV of Q, σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Probability of Failure of of Section			Probability of Occurrence of Failure Mode in Span		Bounds on the PF of the Beam for each Loading Case
		B_1^+ or B_2	C	D_1^+ or D_2	AC	CE	
1+	AC & CE	6.18(-13)	5.23(-8)	6.18(-13)	3.22(-20)	3.22(-20)	$3.22(-20) \leq p_{fs_1} \leq 6.44(-20)$
2	AC	2.71(-10)	3.83(-13)	0	1.03(-22)	0	1.03(-22)
3	CE	0	3.83(-13)	2.71(-10)	0	1.03(-22)	1.03(-22)

Bounds on the probability of failure of the beam : $3.22(-20) \leq p_{fs} \leq 6.46(-20)$

+ Sections B_1 and D_1 correspond to loading case 1.

Table 3.20. Magnitudes of external bending moments (mode values for M_q) at critical sections of the beam of Fig. 3.13 for probabilistic live load.

Sl. No.	Bending Moment due to	Loading of Spans	External Bending Moment (tm) at Section						
			B_1	B_2	C	D	E	F_1	F_2
1	Dead load	AC, CE & EG	5.107	4.8	6.4	1.6	6.4	5.107	4.8
2	Live load	AC, CE & EG	2.488	2.34	3.12	0.78	3.12	2.488	2.34
3	"	AC & EG	-	3.15	1.559	1.559*	1.559	-	3.15
4	"	CE	-	0.78*	1.559	2.335	1.559	-	0.78*
5	"	AC	-	2.85	2.088	0.452*	0.53*	-	0.265
6	"	AC & CE	2.265	-	3.647	1.559	1.027	0.432*	-
7	"	EG	-	0.265	0.53*	0.452*	2.088	-	2.85
8	"	CE & EG	0.432*	-	1.027	1.559	3.647	2.265	-

* These values of M_q are acting opposite to M_g.

Table 3.21. PF of critical sections of the beam of Fig. 3.13 for PV of Q, σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Probability of Failure of Sections									
		B_1^+ or B_2		C		D		E		F_1^+ or F_2	
		UR	OR	UR	OR	UR	OR	UR	OR	UR	OR
1^+	AC, CE & EG	1.1(-11)*	2.1(-11)	2.1(-10)	2.5(-10)	0	0	2.1(-10)	2.5(-10)	1.1(-11)	2.1(-11)
2	AC & EG	1.1(-9)	1.9(-9)	2.1(-16)	6.2(-16)	1.4(-13)	1.1(-12)	2.1(-16)	6.2(-16)	1.1(-9)	1.9(-9)
3	CE	0	0	2.1(-16)	6.2(-16)	2.4(-10)	1.3(-9)	2.1(-16)	6.2(-16)	0	0
4	AC	2.1(-10)	3.9(-10)	1.2(-13)	2.2(-13)	0	0	0	0	0	0
5^+	AC & CE	1.9(-12)	4.0(-12)	2.8(-9)	3.0(-9)	1.4(-13)	1.1(-12)	0	0	0	0
6	EG	0	0	0	0	0	0	1.2(-13)	2.2(-13)	2.1(-10)	3.9(-10)
7^+	CE & EG	0	0	0	0	1.4(-13)	1.1(-12)	2.8(-9)	3.0(-9)	1.9(-12)	4.0(-12)

+ Sections B_1 and F_1 correspond to loading cases 1, 5 and 7.

* 1.1(-11) is read as 1.1×10^{-11}.

Table 3.22. Bounds on the PF of the beam of Fig. 3.13 for PV of Q, σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Probability of Failure of Sections					Probability of Occurrence of Failure Mode in Span			Bounds on the PF of the Beam for each Loading Case
		B_1^+ or B_2	C	D	E	F_1^+ or F_2	AC	CE	EG	
1+	AC, CE & EG	3.2(-11)*	4.6(-10)	0	4.6(-10)	3.2(-11)	1.5(-20)	0	1.5(-20)	$1.5(-20) \leq P_{fs_1} \leq 3.0(-20)$
2	AC & EG	3.0(-9)	8.3(-16)	1.2(-12)	8.3(-16)	3.0(-9)	2.5(-24)	8.3(-43)	2.5(-24)	$2.5(-24) \leq P_{fs_2} \leq 5.0(-24)$
3	CE	0	8.3(-16)	1.5(-9)	8.3(-16)	0	0	1.0(-43)	0	1.0(-43)
4	AC	6.0(-10)	3.4(-13)	0	0	0	2.0(-22)	0	0	2.0(-22)
5+	AC & CE	5.9(-12)	5.8(-9)	1.2(-12)	0	0	3.4(-20)	0	0	3.4(-20)
6	EG	0	0	0	3.4(-13)	6.0(-10)	0	0	2.0(-22)	2.0(-22)
7+	CE & EG	0	0	1.2(-12)	5.8(-9)	5.9(-12)	0	0	3.4(-20)	3.4(-20)

Bounds of the probability of failure of the beam : $3.4(-20) \leq P_{fs} \leq 9.8(-20)$

+ Sections B_1 and F_1 correspond to loading cases 1, 5 and 7.

* 3.2(-11) is read as 3.2×10^{-11}.

Example 3.6

The simply supported PSC beam of Example 3.1 (Fig. 3.11) is considered and the reliability analysis of the same beam, for probabilistic variations of live load, strengths of materials and geometric properties of the section, is illustrated.

It is known from Example 3.5 that the live load is distributed as LN(7.61 tm, 0.368). The value of M_g is equal to 18.75 tm. The following results, obtained by using Monte Carlo method, are taken from Example 3.4.

$$P(O) = 0.07 \quad ; \; P(U) = 0.93$$

$$M_{rum} = 100.51 \text{ tm}; \; s_{ru} = 5.37 \text{ tm}$$

$$M_{rom} = 94.67 \text{ tm} \; ; \; s_{ro} = 17.0$$

Substituting the above values in Eqs. 3.51 and 3.52, the values of $p_{f|u}$ and $p_{f|o}$ are obtained as 1.16×10^{-10} and 1.08×10^{-9} respectively. Using Eq. 3.31, the p_f of the section is

$$p_f = (0.93)\,(1.16 \times 10^{-10}) + (0.07)\,(1.08 \times 10^{-9})$$
$$= 1.84 \times 10^{-10}$$

The results of the reliability analysis of the two span beam of Fig. 3.12 are presented in tables 3.23 and 3.24 and of the three span beam of Fig. 3.13 in tables 3.25 and 3.26 for the probabilistic variations of Q, σ_{cu}, σ_s and dimensions of section.

Table 3.23. PF of critical sections of the beam of Fig. 3.12 for PV of Q, σ_{cu}, σ_s and dimensions of section

Sl. No.	Live Load on Spans	Probability of Failure of Section					
		B_1^+ or B_2		C		D_1^+ or D_2	
		UR	OR	UR	OR	UR	OR
1+	AC & CE	3.95(-14)*	2.2(-14)	4.97(-9)	1.56(-9)	3.95(-14)	2.2(-14)
2	AC	2.67(-11)	1.22(-11)	1.82(-14)	1.09(-14)	0	0
3	CE	0	0	1.82(-14)	1.09(-14)	2.67(-11)	1.22(-11)

Table 3.24. Bounds on the PF of the beam of Fig. 3.12 for PV of Q, σ_{cu}, σ_s and dimensions of section

Sl. No.	Live Load on Spans	Probability of Failure of Section			Probability of Occurrence of Failure Mode in Span		Bounds on the PF of the Beam for each Loading Case
		B_1^+ or B_2	C	D_1^+ or D_2	AC	CE	
1+	AC & CE	6.15(-14)	6.53(-9)	6.15(-14)	3.82(-22)	3.82(-22)	$3.82(-22) \leq P_{fs_1} \leq 7.64(-22)$
2	AC	3.89(-11)	2.91(-14)	0	1.13(-24)	0	1.13(-24)
3	CE	0	2.91(-14)	3.89(-11)	0	1.13(-24)	1.13(-24)

Probability of failure of the beam : $3.82(-22) \leq P_{fs} \leq 7.66(-22)$

+ Sections B_1 and D_1 correspond to loading case 1.

* 3.95(-14) is read as 3.95×10^{-14}.

Table 3.25. PF of critical sections of the beam of Fig. 3.13 for PV of Q, σ_{cu} , σ_s and dimensions of section

Sl. No.	Live Load on Span	Probability of Failure of Sections									
		B_1^+ or B_2		C		D		E		F_1^+ or F_2	
		UR	OR	UR	OR	UR	OR	UR	OR	UR	OR
1+	AC, CE & EG	2.1(-12)*	1.4(-12)	4.1(-11)	1.8(-11)	0	0	4.1(-11)	1.8(-11)	2.1(-12)	1.4(-12)
2	AC & EG	2.4(-10)	1.5(-10)	1.8(-17)	2.6(-17)	2.6(-14)	8.5(-14)	1.8(-17)	2.6(-17)	2.4(-10)	1.5(-10)
3	CE	0	0	1.8(-17)	2.6(-17)	8.1(-11)	1.3(-10)	1.8(-17)	2.6(-17)	0	0
4	AC	4.2(-11)	2.9(-11)	1.9(-14)	1.2(-14)	0	0	0	0	0	0
5+	AC & CE	3.4(-13)	2.6(-13)	6.1(-10)	2.3(-10)	2.6(-14)	8.5(-14)	0	0	0	0
6	EG	0	0	0	0	0	0	1.9(-14)	1.2(-14)	4.2(-11)	2.9(-11)
7+	CE & EG	0	0	0	0	2.6(-14)	8.5(-14)	6.1(-10)	2.3(-10)	3.4(-13)	2.6(-13)

+ Sections B_1 and F_1 correspond to loading cases 1, 5 and 7.

* 2.1(-12) is read as 2.1×10^{-12}.

Table 3.26. Bounds on the PF of the beam of Fig. 3.13 for PV of Q, σ_{cu}, σ_s and dimensions of section

Sl. No.	Live Load on Spans	Probability of Failure of Sections					Probability of Occurrence of Failure Mode in Span			Bounds on the PF of the Beam for each Loading Case
		B_1^+ or B_2	C	D	E	F_1^+ or F_2	AC	CE	EG	
1+	AC, CE & EG	3.5(-12)*	5.9(-11)	0	5.9(-11)	3.5(-12)	2.1(-22)	0	2.1(-22)	$2.1(-22) \leq p_{fs_1} \leq 4.2(-22)$
2	AC & EG	3.9(-10)	4.4(-17)	1.1(-13)	4.4(-17)	3.9(-10)	1.7(-26)	2.1(-46)	1.7(-26)	$1.7(-26) \leq p_{fs_2} \leq 3.4(-26)$
3	CE	0	4.4(-17)	2.1(-10)	4.4(-17)	0	0	4.1(-43)	0	4.1(-43)
4	AC	7.1(-11)	3.1(-14)	0	0	0	2.2(-24)	0	0	2.2(-24)
5+	AC & CE	6.0(-13)	8.4(-10)	1.1(-13)	0	0	5.0(-22)	0	0	5.0(-22)
6	EG	0	0	0	3.1(-14)	7.1(-11)	0	0	2.2(-24)	2.2(-24)
7+	CE & EG	0	0	1.1(-13)	8.4(-10)	6.0(-13)	0	0	5.0(-22)	5.0(-22)

Bounds on the probability of failure of the beam : $5.0(-22) \leq p_{fs} \leq 14.2(-22)$

+ Sections B_1 and F_1 correspond to loading cases 1, 5 and 7.

* 3.5(-12) is read as 3.5×10^{-12}.

CHAPTER 4

RELIABILITY ANALYSIS OF PRESTRESSED CONCRETE BEAMS AT LIMIT STATES OF CRACKING AND DEFLECTION

4.1 GENERAL

A structure is said to have failed if it has lost its resistance against load or has become unserviceable because of undesirable deformation or cracking etc. The limit states can be placed in two categories:

(a) the ultimate limit state which corresponds to the maximum load carrying capacity;

(b) the serviceability limit states which are related to the criteria governing normal use and durability.

The causes leading to the attainment of serviceability limit states are

(i) excessive deformations with respect to normal use of structure

(ii) premature or excessive cracking

(iii) undesirable damage (corrosion)

(iv) excessive displacement without loss of equilibrium

(v) excessive vibrations, etc.

Limit states of initial crack formation and deflection are considered in this thesis. Reliability analysis of simply supported and continuous prestressed concrete beams at limit

state of initial crack formation is first presented for deterministic and probabilistic loads considering probabilistic variations of:

(i) strengths of concrete and steel

(ii) strengths of concrete and steel and geometric properties of the section.

Reliability analysis at limit state of deflection is presented for the probabilistic variations of:

(i) strengths of materials

(ii) strengths of materials and live load

(iii) strengths of materials, live load and geometric properties of the section.

4.2 RELIABILITY ANALYSIS AT LIMIT STATE OF CRACKING

4.2.1 Introduction

Codes specify that prestressed concrete beams possess certain minimum resisting moment at the limit state of cracking which can be defined as the state when the maximum tensile stress in the extreme fibre due to external moment reaches the value of modulus of rupture of concrete. This external moment producing first crack in a prestressed concrete beam is computed by the elastic theory assuming that cracking starts when the modulus of repture of concrete is exceeded. Defining M_{rc} as the resisting moment of the section at initial crack formation, the beam is said to become unserviceable when the external moment exceeds the value of M_{rc}. Hence the

probability of cracking of the section, p_c, can be written as

$$p_c = P(M_e > M_{rc}) \tag{4.1}$$

M_{rc} is a function of several random variables of strengths of materials and geometric properties of the section. The probability distribution and parameters of M_{rc} are best obtained by Monte Carlo technique, explained in the previous chapter.

4.2.2 Equations for the Determination of M_{rc}

The following equations, to determine M_{rc} of a section, are necessary in the Monte Carlo method. It is assumed that the cables are fully bonded and so the transformed area of cross section is used in the calculation of M_{rc}. The state of stress and strain at limit state of cracking is shown in Fig. 4.1. If E_s is the Young's Modulus of steel, ϵ_{ct} is the tensile strain at initial cracking, m is the modular ratio and σ_r is the modulus of rupture of concrete, the depth of neutral axis, x, is given by

$$x = \frac{r_1 + r_2(r_3 h + r_4 d)}{2r_5 + r_2(r_3 + r_4)} \tag{4.2}$$

where

$$r_1 = t_t^2 (b_t - b') - t_b^2 (b_b - b') + 2ht_b(b_b - b') + 2r_6 d + b'h^2$$

$$r_2 = 2A_{ts}/\sigma_r$$

$$r_3 = 0.66 A_{ts}$$

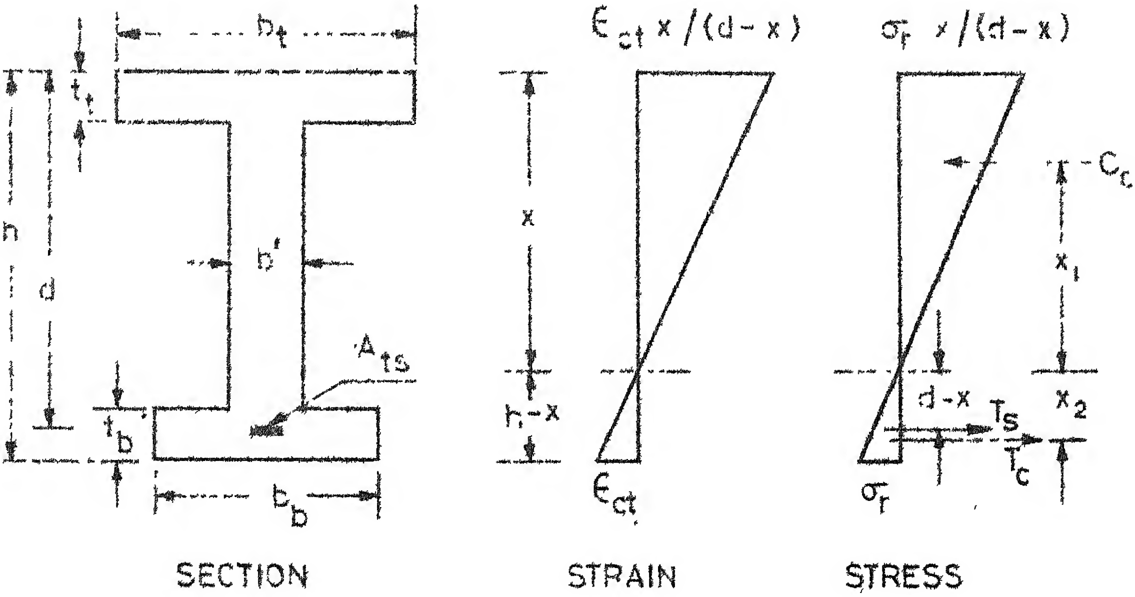


FIG 4.1 STATE OF STRESS AND STRAIN AT LIMIT STATE OF CRACKING

$$r_4 = E_s \varepsilon_{ct}$$

$$r_5 = t_t(b_t - b') + t_b(b_b - b') + r_6 + b'h$$

$$r_6 = (m-1)\ \bar{A}_{ts}$$

b_b denotes breadth of bottom flange and t_b, the thickness of bottom flange. The Eq. 4.2 is valid whether the neutral axis lies in the web or bottom flange.

Case 1. Neutral axis in the web:

The following equations are used to determine M_{rc} if the neutral axis lies in the web.

The coopressive force in concrete, C_c , is

$$C_c = [b_t x^2 - (b_t - b')\ (x-t_t)^2]\ \frac{\sigma_r}{2(h-x)} \tag{4.3a}$$

which acts at a distance

$$x_1 = \frac{2}{3}\ [\frac{b_t x^3 - (b_t - b')\ (x-t_t)^3}{b_t x^2 - (b_t - b')\ (x-t_t)^2}] \tag{4.4a}$$

from the neutral axis. The tensile force in concrete, T_c, is given by

$$T_c = [\bar{b}_b(\bar{h}-\bar{x})^2 - (\bar{b}_b - \bar{b}')\ (h-x-\bar{t}_b)^2 + 2r_6(d-\bar{x})\ ]\ \frac{\sigma_r}{2(h-x)} \tag{4.5a}$$

which acts at a distance

$$x_2 = \frac{2}{3}\ [\frac{b_b(h-x)^3 - (b_b - b')(h-x-t_b)^3 + 3r_6(d-x)^2}{b_b(h-x)^2 - (b_b - b')(h-x-t_b)^2 + 2r_6(d-x)}] \tag{4.6a}$$

from the neutral axis. Assuming ε_{ce} is equal to one tenth of ε_{se} and effective tensile stress in prestress steel to the maximum

value of 0.6 σ_s(46), the strain in steel, ε_{sc}, at initial cracking is obtained from the equation

$$\varepsilon_{sc} = \left(\frac{0.66\ \sigma_s}{E_s}\right) + \varepsilon_{ct}\left(\frac{d-x}{h-x}\right) \tag{4.7}$$

The total tensile force in steel, T_s, is

$$T_s = \varepsilon_{sc}\, E_s\, A_{ts} \tag{4.8}$$

which acts at a distance (d-x) from the neutral axis. Taking moments of C_c, T_c and T_s about neutral axis, M_{rc}, is given by

$$M_{rc} = C_c\, x_1 + T_c x_2 + T_s\,(d-x) \tag{4.9}$$

Case 2. Neutral axis in bottom flange:

The following equations are used to determine M_{rc} if the neutral axis lies in the bottom flange.

$$C_c = [(b_t-b')t_t(2x-t_t)+(b_b-b')(x-h+t_b)^2+b'x^2]\ \frac{\sigma_r}{2(h-x)} \tag{4.3b}$$

$$x_1 = \frac{2}{3}\left[\frac{(b_b-b')(x-h+t_b)^3+b'x^3+(b_t-b')t_t(3x^2-3xt_t+t_t^2)}{(b_b-b')(x-h+t_b)^2+b'x^2+(b_t-b')t_t(2x-t_t)}\right] \tag{4.4b}$$

$$T_c = [\,b_b(h-x)^2 + 2r_6(d-x)\,]\ \frac{\sigma_r}{2(h-x)} \tag{4.5b}$$

$$x_2 = \frac{2}{3}\left[\frac{b_b(h-x)^3 + 3r_6(d-x)^2}{b_b(h-x)^2 + 2r_6(d-x)}\right] \tag{4.6b}$$

Values of ε_{sc}, T_s and M_{rc} are given by Eqs. 4.7, 4.8 and 4.9 respectively.

The above equations are functions of material properties ε_{ct}, σ_r, m and σ_s and geometric properties b_t, b_b, t_t, t_b, b', d, h and A_{ts} of the section. As elastic theory is valid upto initial crack formation (58)

$$\varepsilon_{ct} = \frac{\sigma_r}{E_c} \tag{4.10}$$

where E_c is the Young' Modulus of concrete which is given by (46)

$$E_c = 18000 \sqrt{\sigma_{cu}} \tag{4.11}$$

The value of σ_r is equal to

$$\sigma_r = 2.25 \sqrt{\sigma_{cu}} \tag{4.12}$$

Hence σ_r and E_c can be connected to the strength of concrete whose probability distribution and parameters are known. A_{ts} can be connected to diameter of steel whose probability distribution is also known. It is assumed that random variations of b_b and t_b are same as b_t and t_t. Hence probability distribution and parameters of all random variables are known.

4.2.3 Probability Distribution of M_{rc} for Probabilistic Variations of Strengths of Materials

It can be seen from Eq. 4.9 that M_{rc} is a function of several random variables. The analysis is first presented considering probabilistic variations of σ_{cu} and σ_s which are distributed as N(422.8,56) kg/cm^2 and N(15680, 488) kg/cm^2 respectively (Refer Chapter 2). Using Monte Carlo method, explained in article 3.3,

10000 samples have been generated for M_{rc} of the section shown in Fig. 3.11. The histogram and cumulative distribution of the generated data are shown in Fig. 4.2. The skewness is very very small and normal distribution is found to satisfy the chi square test at one percemt level of significance. Hence normal distribution is adopted for M_{rc} of section for probabilistic variations of σ_{cu} and σ_s.

4.2.4 Computation of Probability of Cracking for Deterministic Load

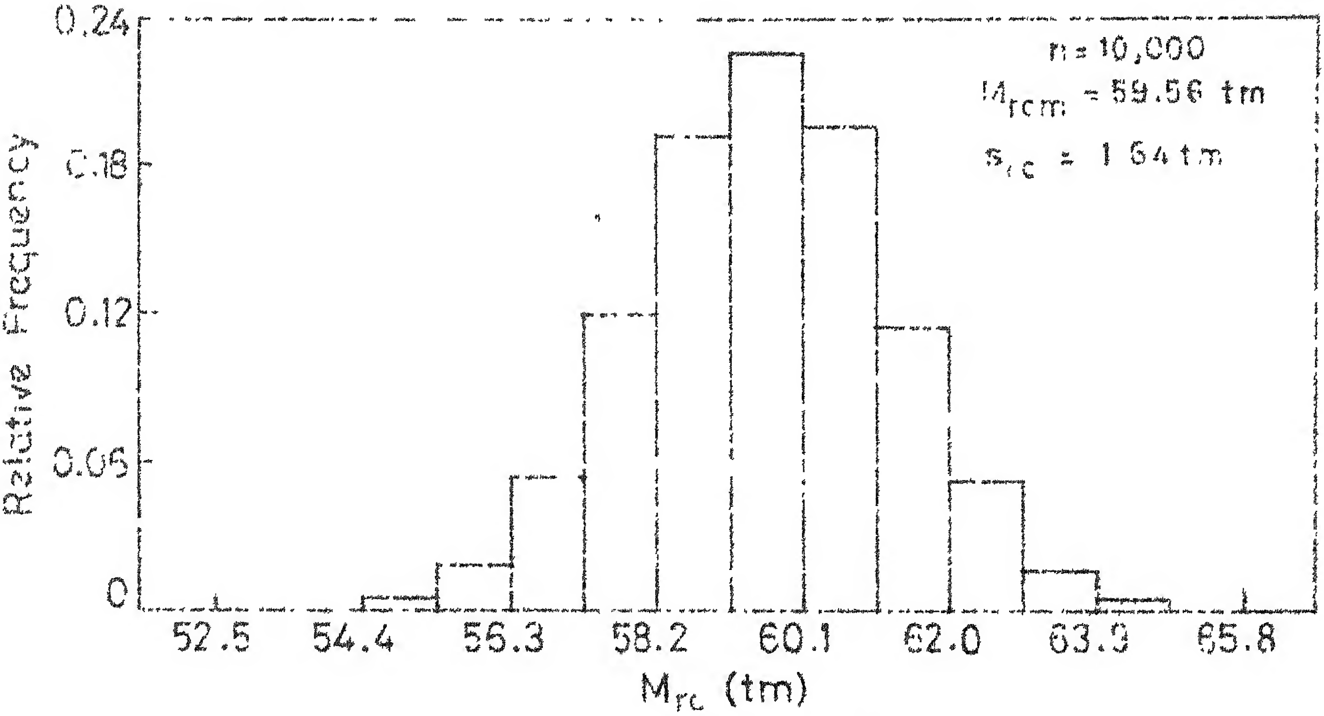
The probability of cracking of a section has been defined by Eq. 4.1. For normally distributed M_{rc}, the probability of cracking is given by

$$p_c = P(M_{rc} < M_{ew}) = \phi\left(\frac{M_{ew} - M_{rcm}}{s_{rc}}\right) \qquad (4.13)$$

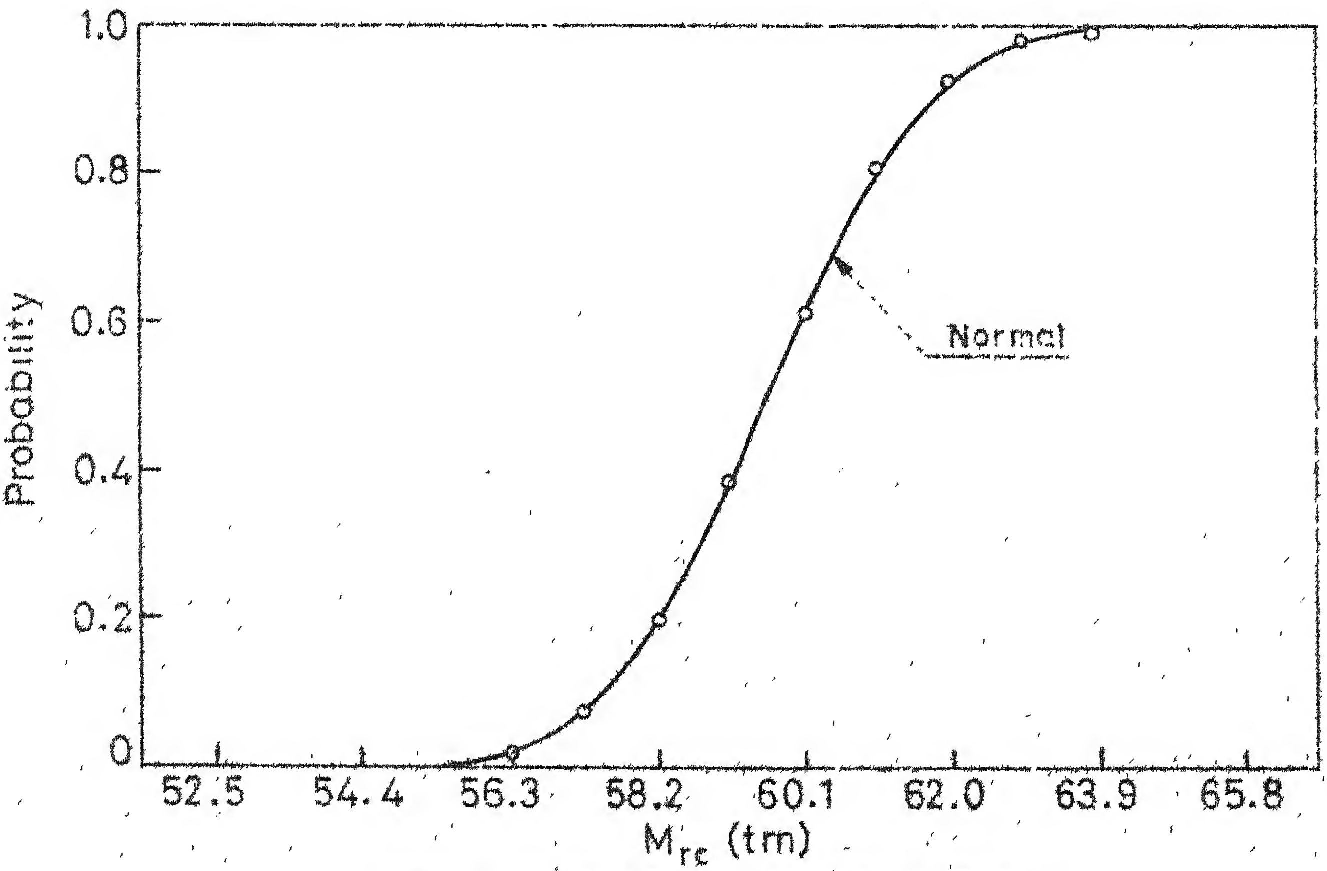
where M_{rcm} and s_{rc} are the parameters mean and standard deviation of M_{rc}. The method of reliability analysis at limit state of cracking is illustrated with an example.

Example 4.1

Simply supported PSC beams, supporting a floor, are spaced at 5m. Live load on the floor is 400 kg/m^2. Total dead load (including self-weight of the beam) on any intermediate beam is 1500 kg/m. The beams are designed with concrete cube strength of 350 kg/cm^2 and steel strength of 15000 kg/cm^2. The effective span and area of steel are 10m and 11.83 cm^2 respectively. The



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG. 4.2 VARIATION OF M_{rc} FOR PV OF σ_{cu} AND σ_s

cross section of the beams and cable profile are shown in Fig. 3.11 which is again given here for ready reference. The beams are designed assuming they are cast separately and the composite action of slab and beam is not considered.

One of the intermediate beams is considered and the reliability analysis of the same at limit state of cracking is illustrated.

For this beam, using Monte Carlo method parameters of M_{rc} have already been obtained in article 4.23 and given in Fig. 4.2. Their values are

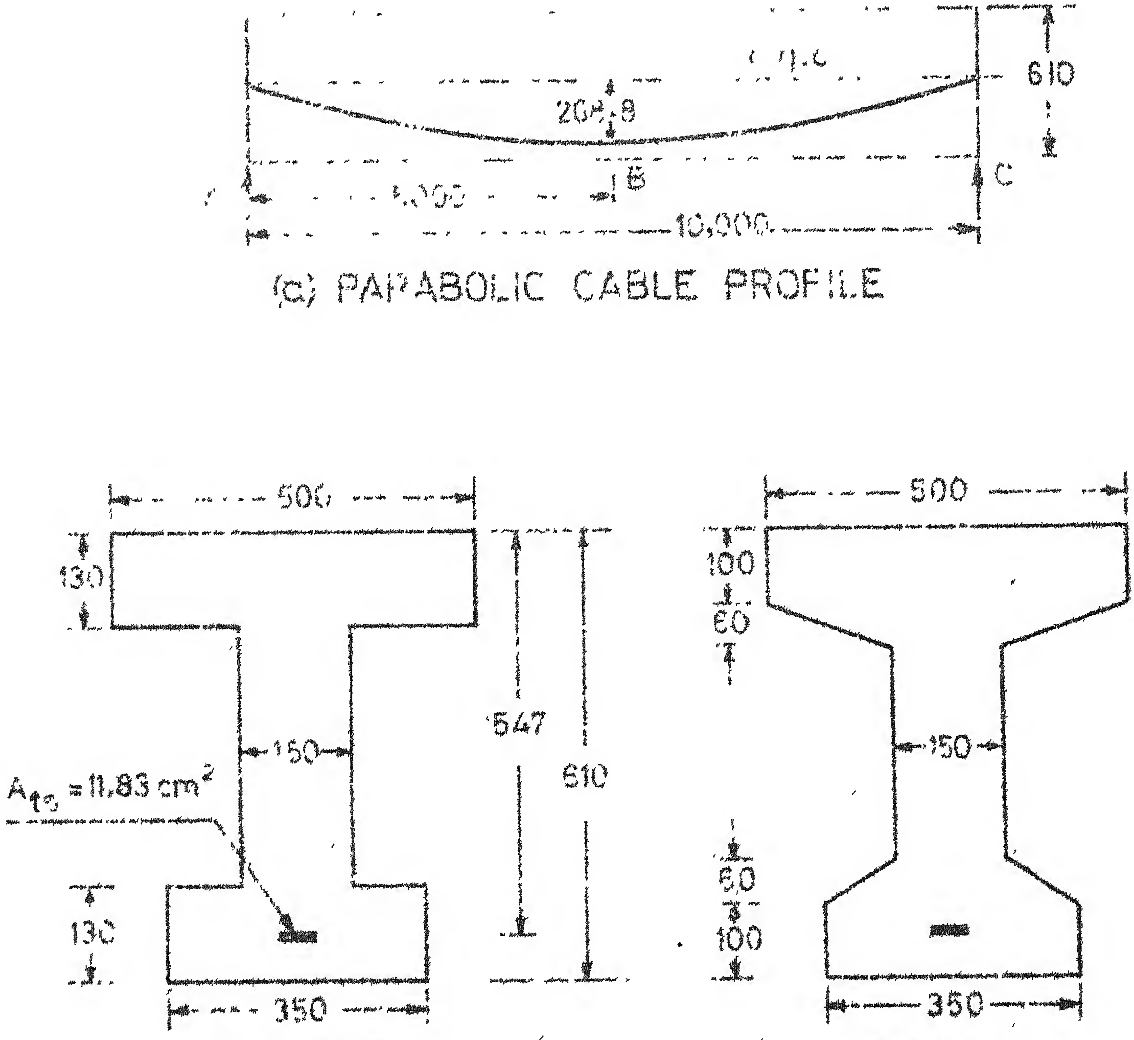
$$\bar{M}_{rc} = 59.56 \text{ tm} \; ; \; s_{rc} = 1.64 \text{ tm}$$

The external bending moment at midspan, from Example 3.1, is 43.75 tm. Using Eq. 4.13, the probability of cracking of the beam is

$$p_c = \phi\left(\frac{43.75 - 59.56}{1.64}\right) = 2.7 \times 10^{-22}$$

4.2.5 Reliability Analysis of Continuous PSC Beams at Limit State of Cracking

Continuous beams are analysed for different possible live load conditions. For each loading case, the critical section where probability of failure is maximum, is determined. For deterministic load, the critical section depends upon the difference between the mean values of M_{ew} and M_{rc} and s_{rc} of each section. The value of M_{rc} of a section is approximately proportional to the eccentricity of cable at that section. The value of s_{rc} is assumed approximately equal to 0.9 s_s(observed later) for probabilistic



(c) PARABOLIC CABLE PROFILE

IDEALISED

ACTUAL

(b) MIDSPAN SECTION

FIG.3.11 SIMPLY SUPPORTED BEAM-EXAMPLE 3.1

ALL DIMENSIONS ARE IN mm

variations of σ_{cu} and σ_s. The equation for M_{ew} of a section is known. Hence critical section can be fixed approximately. The probability of cracking of the critical section for each loading case can be computed by Eq. 4.13. If p_{ci} is the probability of cracking of the beam for loading condition i, the bounds on the value of p_{cs} of the beam for n independent load conditions is given by (26)

$$\text{Max } p_{ci} \leq p_{cs} \leq \sum_{i=1}^{n} p_{ci} \qquad (4.14)$$

where p_{cs} is the probability of cracking of the beam for n load conditions. The method is illustrated with examples.

Example 4.2

Continuous PSC beams, having two equal spans and supporting a floor, are spaced at 5 m. Live load on the floor is 400 kg/m^2. Total dead load (including self-weight of the beam) on any intermediate beam is 1500 kg/m. The beams are designed with concrete cube strength of 350 kg/cm^2 and steel strength of 15000 kg/cm^2. The effective span and area of steel are 10m and 10.89 cm^2. The beams have uniform cross section which is shown in Fig. 3.12. The same figure is again given here for ready reference. The beams are designed assuming that they are cast separately and the composite action of slab and beam is not considered.

One of the intermediate beams only is considered and the reliability analysis of the same at limit state of cracking is illustrated.

There are three critical live load conditions on the beam:

(i) Live load on both spans

(ii) Live load on span AC

(iii) Live load on span CE

Using the method explained in article 4.2.5, the critical sections of cracking are C, B_2 and D_2 (Fig. 3.12) for loading conditions (i), (ii) and (iii) respectively. The external moments, mean values and standard deviations of σ_{cu} and σ_s remain same as in Example 3.2. Using the parameters and distributions of σ_{cu} and σ_s, 5000 samples are generated for M_{rc} of each critical section by Monte Carlo method. The results are given below.

Section B_2 or D_2

$$M_{rcm} = 46.79 \text{ tm}; \; s_{rc} = 1.31 \text{ tm}$$

Section C

$$M_{rcm} = 51.60 \text{ tm}; \; s_{rc} = 1.42 \text{ tm}$$

Load condition (i) : Live load on both spans

Critical section is C. From table 3.2, the value of M_{ew} at this section is 43.75 tm. Using Eq. 4.13, the probability of cracking of the beam for the loading condition (i) is

$$p_{c1} = \phi\left(\frac{43.75 - 51.6}{1.42}\right) = 1.6 \times 10^{-8}$$

Load condition (ii): Live load on span AC

Critical section is B_2. From table 3.2, the value of M_{ew} at this section is 28.125 tm. Using Eq. 4.13, the probability of

cracking of the beam for the loading condition (ii), p_{c2}, is found to be zero.

Similarly, when live load is on the span CE, p_{c3} can be computed and found to be equal to zero.

Maximum value of the probability of cracking is for the loading condition (i). Hence, the bounds on the value of p_{cs}, using Eq. 4.14, is

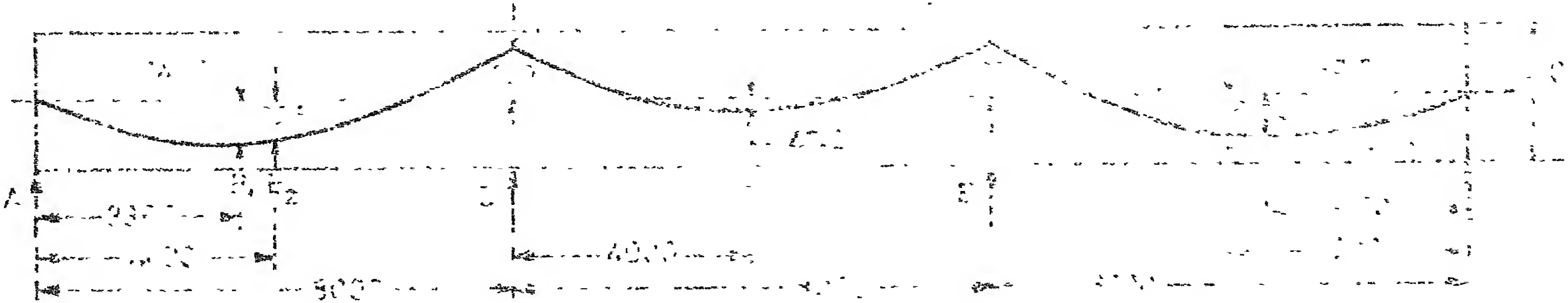
$$p_{c1} \leq p_{cs} \leq p_{c1} + p_{c2} + p_{c3}$$

As the values of p_{c2} and p_{c3} are equal to zero, the value of p_{cs} is equal to p_{c1}. Hence

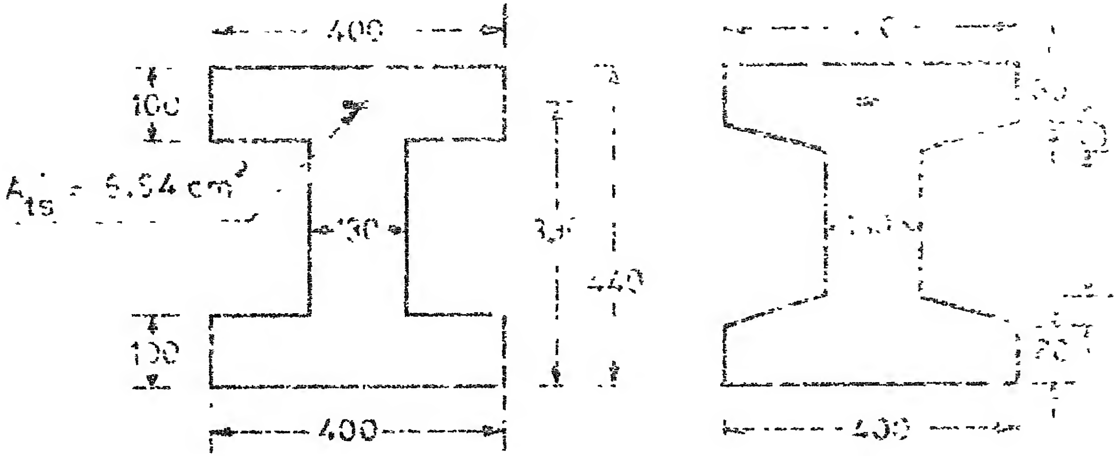
$$p_{cs} = 1.6 \times 10^{-8}$$

Example 4.3

Continuous PSC beams, having three equal spans and supporting a floor, are spaced at 4m. Live load on the floor is 400 kg/m^2. Total dead load (including self-weight of the beam) on any intermediate beam is 1000 kg/m. The beams are designed with concrete strength of 350 kg/cm^2 and steel strength of 15000 kg/cm^2. The effective span and area of steel are 8m and 6.94 cm^2 respectively. The beams have uniform cross section which is shown in Fig. 3.13. The same figure is again given here for ready reference. The beams are designed assuming that they are cast separately and the composite action of slab and beam is not considered.



(a) PARABOLIC CABLE PROFILE



IDEALISED

ACTUAL

(b) SECTION AT C

FIG.3.13 THREE SPAN CONTINUOUS BEAM - EXAMPLE 3.3

ALL DIMENSIONS ARE IN mm

One of the intermediate beams only is considered and the reliability analysis of the same at limit of cracking is illustrated.

There are seven different loading conditions of the beam for live load. They are listed in table 4.1. Critical section for each loading case is also shown in the same table. The external bending moments, mean values and standard deviations of σ_{cu} and σ_s remain same as in Example 3.3. Using the parameters of σ_{cu} and σ_s and their distributions, 5000 samples are generated for M_{rc} of each critical section by Monte Carlo method. The results are given in table 4.1.

Loading condition (i) : Live load on all spans

Critical section is C. From table 3.5, the value of M_{ew} at C is equal to 16.64 tm. From table 4.1, the value of M_{rcm} of the section C is 25.39 tm. Using Eq. 4.13, the value of p_{c1} is

$$p_{c1} = \phi\left(\frac{16.64 - 25.39}{0.709}\right) = 0$$

Similarly the values of p_{ci}, shown in table 4.1, can be calculated for other loading conditions. Using Eq. 4.14 and the values of p_{ci} from table 4.1, the bounds on p_{cs} are obtained as

$$2.08 \times 10^{-23} \leq p_{cs} \leq 0 + 0 + 0 + 0 + 2.08 \times 10^{-23} + 0 + 2.08 \times 10^{-23}$$

$$2.08 \times 10^{-23} \leq p_{cs} \leq 4.16 \times 10^{-23}$$

Table 4.1. Parameters of M_{rc} and probability of cracking of the beam (shown in Fig. 3.13) for each loading case for PV of σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Critical Section	M_{ew} (tm)	M_{rcm} (tm)	s_{rc} (tm)	p_{ci}
1	AC, CE & EG	C or E	16.64	25.39	0.709	0
2	AC & EG	B_2 or F_2	15.43	22.29	0.646	0
3	CE	D	9.27	15.68	0.508	0
4	AC	B_2	14.67	22.29	0.646	0
5	AC & CE	C	18.38	25.39	0.709	2.08(-23)*
6	EG	F_2	14.67	22.29	0.646	0
7	CE & EG	E	18.38	25.39	0.709	2.08(-23)

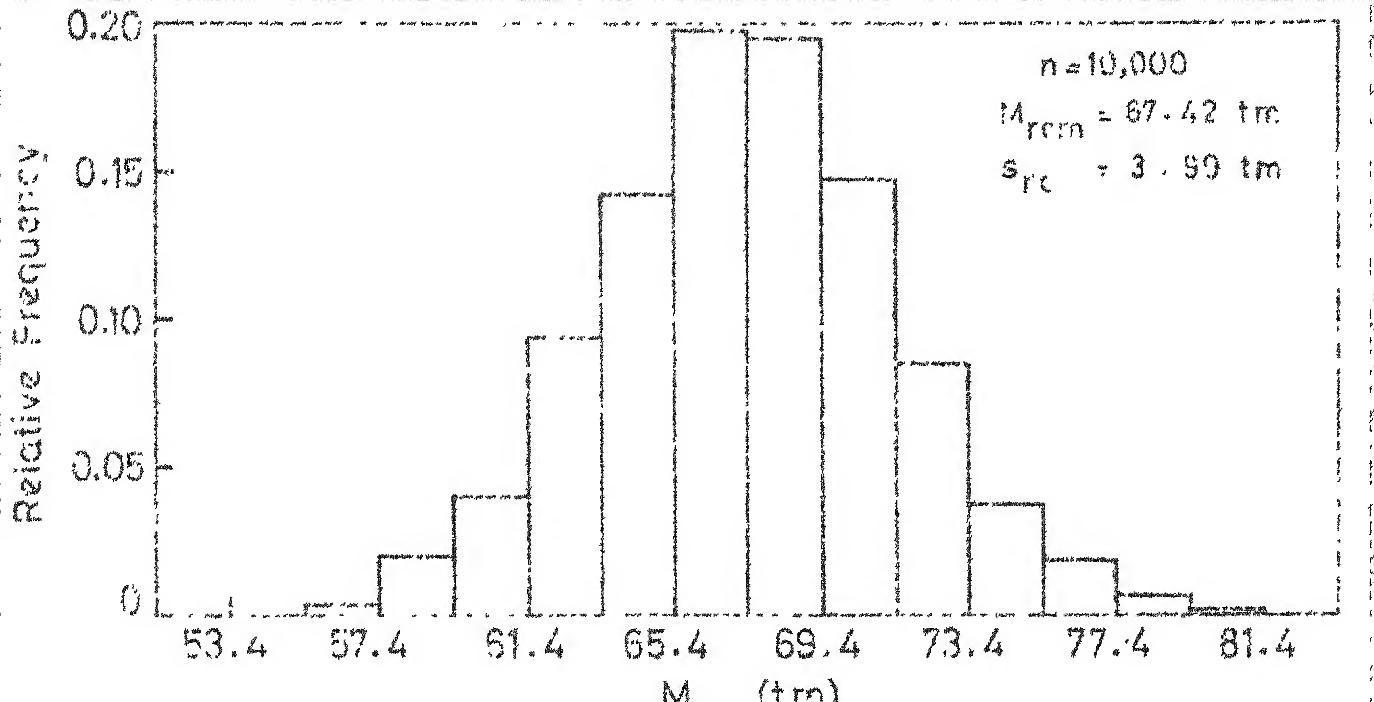
Bounds on p_{cs} of the beam : $2.08(-23) \leq p_{cs} \leq 4.16(-23)$

* 2.08(-23) is read as 2.08×10^{-23}.

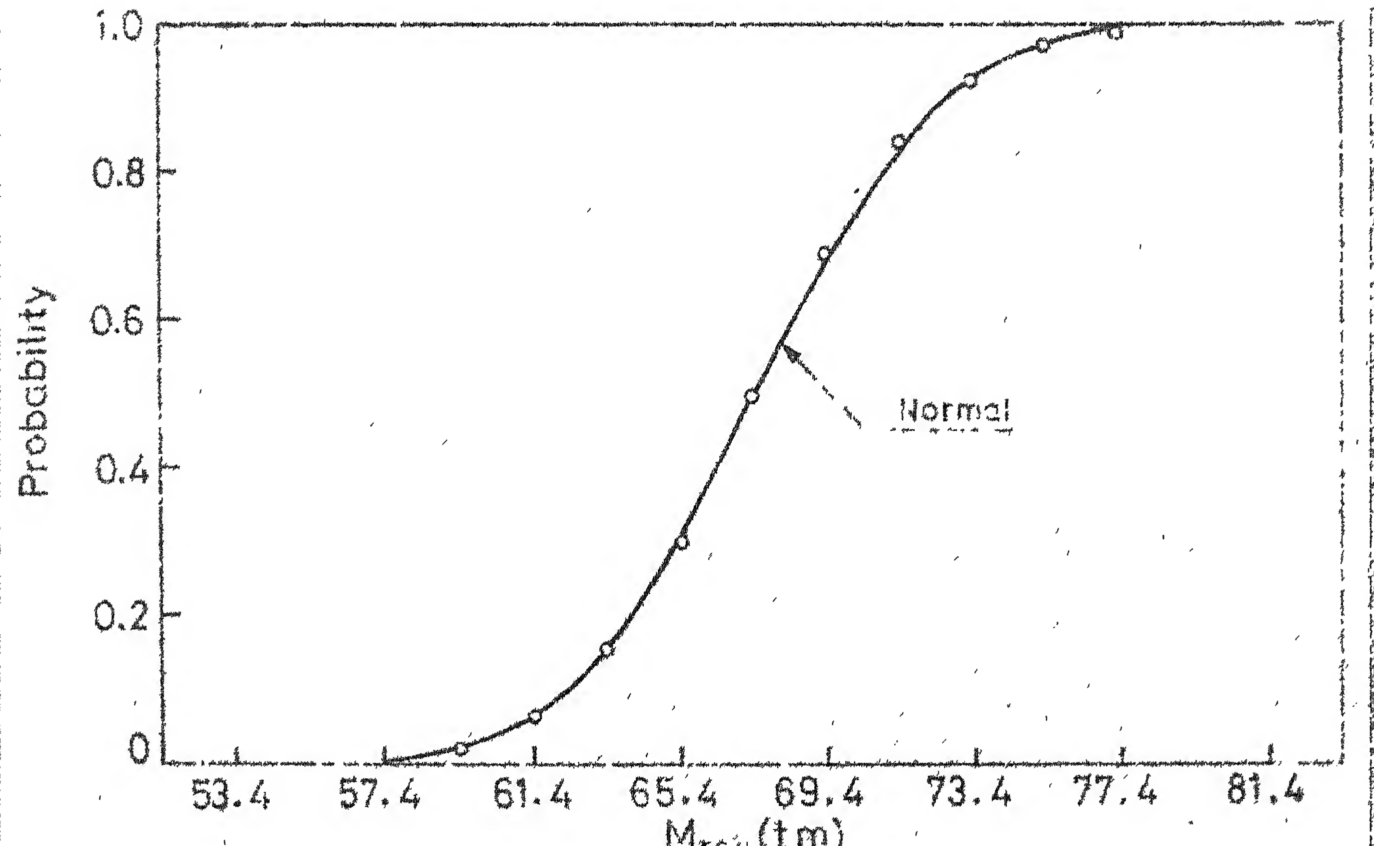
4.2.6 Probability Distribution of M_{rc} for Probabilistic Variations of σ_{cu}, σ_s and Dimensions of Section

Probabilistic variations of σ_{cu} and σ_s only have been considered in the article 4.2.3 to determine the probability distribution and parameters of M_{rc}. It has been observed in Chapter 2 that b_t, b', t_t, d, h and D_s have also random variations. Hence to be more realistic in analysis, the random variations of above parameters are also considered in the equations to determine M_{rc}. The section shown in Fig. 3.11 is considered. The mean values and standard deviations of σ_{cu}, σ_s, b_t, b', t_t, d, h and D_s of the section are taken from Example 3.4 and of b_b and t_b are assumed same as b_t and t_t. Using those values and distributions of above random variables, 10000 samples are generated for M_{rc} of the section by Monte Carlo method. The histogram and cumulative distribution of the generated data are shown in Fig. 4.3. Normal distribution does not satisfy the chi square test for one percent level of significance. However it satisfies at 0.1 percent level. The coefficient of skewness is very small (less than 0.07) and the coefficient of kurtosis is 2.98 which is very close to the value of 3 for true normal. Hence normal distribution is adopted for M_{rc} for probabilistic variations of σ_{cu}, σ_s and dimensions of section.

Having found out the probability distribution of M_{rc}, the reliability analysis of beams at limit state of cracking is done as follows:



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG. 4.3 VARIATION OF M_{rc} FOR PV OF σ_{cu}, σ_s AND DIMENSIONS OF SECTION

(i) Determine the critical section of the beam.

(ii) Using normal distributions and parameters of σ_{cu}, σ_s, b_t, b_b, t_t, t_b, b', d, h and D_s, generate samples for M_{rc} of the critical section by Monte Carlo method. Sample mean and standard deviation of the genrated data will be the parameters of M_{rc}.

(iii) Calculate M_{ew} at the critical section.

(iv) Compute p_c using Eq. 4.13.

The procedure is illustrated with examples

Example 4.4

The simply supported PSC beam of Example 4.1 (Fig. 3.11) is considered. Dimensions of the section and strengths of materials are subjected to random variations.

The critical section is the midspan of the beam. The value of M_{ew} at the critical section is 43.75 tm. Using Monte Carlo technique, the values of M_{rcm} and s_{rc} have already been found in the previous article for this beam. From Fig. 4.3,

M_{rcm} = 67.42 tm ; s_{rc} = 3.99 tm

Using Eq. 4.13, the value of p_c is found to be 1.5×10^{-9}.

The reliability analysis of continuous beams at limit state of cracking and for probabilistic variations of σ_{cu}, σ_s and dimensions of section remain same as explained in the article 4.2.5 except that probabilistic variations of b_t, b_b, t_t, t_b, b', d, h

and D_s are also to be considered in determining the parameters of M_{rc}. The results of the reliability analysis of the two span beam of Fig. 3.12 are given in table 4.2 and for the three span beam of Fig. 3.13 in table 4.3.

4.2.7 Computation of p_c for Probabilistic Load

It is known from Chapter 2 that live load is distributed as LN(121.77 kg/m^2,0.368). The dead load is assumed as deterministic as in the previous chapter. The resistance of the section at initial crack formation is normally distributed as determined in articles 4.2.4 and 4.2.6. When the load and resistance are lognormally and normally distributed, the probability of cracking is calculated by using Eq. 3.51. Using the above equation, the value of p_c for probabilistic load is given by

$$p_c = \int_{-\infty}^{\infty} e^{-w^2} \frac{1}{\sqrt{\pi}} \left[1 - \phi\left\{\frac{\log\left(\frac{\sqrt{2}w\, s_{rc} + M_{rcm} - M_g}{M_{qm}}\right)}{s_q}\right\}\right] dw \tag{4.15}$$

The reliability analysis of PSC beams at limit state of cracking, considering probabilistic variation of Q, is illustrated with examples. In all the following examples in this section, live load is distributed as LN(121.77 kg/m^2, 0.368).

Example 4.5

The simply supported PSC beam of Example 4.1 (Fig. 3.11) is subjected to probabilistic variations of live load and strengths

Table 4.2. Parameters of M_{rc} and probability of cracking of the beam in Fig. 3.12 for each loading case for PV of σ_{cu} , σ_s and dimensions of section

Sl. No.	Live Load on Spans	Critical Section	M_{ew} (tm)	M_{rcm} (tm)	s_{rc} (tm)	p_{ci}
1	AC and CE	C	43.75	58.31	3.36	7.5(-6)*
2	AC	B_2	28.25	52.91	3.03	1.98(-16)
3	CE	D_2	28.25	52.91	3.03	1.98(-16)

Bounds on p_{cs} of the beam : $7.5(-6) \leq p_{cs} \leq 7.5(-6)$

* 7.5(-6) is read as 7.5×10^{-6}.

Table 4.3. Parameters of M_{rc} and probability of cracking of the beam in Fig, 3.13 for each loading case for PV of σ_{cu} , σ_s and dimensions of section

Sl. No.	Live Load on Spans	Critical Section	M_{ew} (tm)	M_{rcm} (tm)	s_{rc} (tm)	p_{ci}
1	AC, CE & EG	C or E	16.64	28.73	1.607	2.5(-14)
2	AC & EG	B_2 or F_2	15.43	25.24	1.457	9.0(-12)
3	CE	D	9.27	17.85	1.046	1.1(-16)
4	AC	C	14.67	25.24	1.457	1.8(-13)
5	AC & CE	B_2	18.38	28.73	1.607	6.0(-11)
6	EG	E	14.67	25.24	1.457	1.8(-13)
7	CE & EG	F_2	18.38	28.73	1.607	6.0(-11)

Bounds on p_{cs} of the beam : $6.0(-11) \leq p_{cs} \leq 12.93(-11)$.

of materials. Other particulars of the beam remain same as in Example 4.1

From Example 4.1, the parameters of M_{rc} of the midspan of the beam are

$$M_{rcm} = 59.56 \text{ tm} \; ; \; s_{rc} = 1.64 \text{ tm}$$

From Example 3.7,

$$M_g = 18.75 \text{ tm} \; ; \; M_{qm} = 7.61 \text{ tm.}$$

Substituting the values of M_{rcm}, s_{rc}, M_g and M_{qm} in Eq. 4.15 and solving the same by using Gaussian Quadrature formula (explained in the previous chapter), the value of p_c is found to be 2.94×10^{-6}.

The reliability analysis of continuous beams at limit state of cracking for probabilistic variations of Q, σ_{cu} and σ_s remain same as explained in the article 4.2.5 except that p_c is evaluated using Eq. 4.15.

The same two span and three span continuous PSC beams of Examples 4.2 and 4.3 are considered. The values of M_g and M_{qm} at the critical sections of the beam of Example 4.2 are available in table 3.17 and parameters of M_{rc} in Example 4.2. The values of M_g and M_{qm} at the critical sections of the beam of Example 4.3 are available in table 3.20 and parameters of M_{rc} in Example 4.3. The results of the reliability analysis of the above beams at limit state of cracking are presented in tables 4.4 and 4.5.

Example 4.6

The simply supported PSC beam of Example 4.1 is subjected to probabilistic variations of live load, strengths of materials and dimensions of the section. The live load is distributed as LN(121.77 kg/m^2,0.368). Other particulars of the beam remain same as in Example 4.1.

Using Monte Carlo method, the parameters of M_{rc} of the midspan of the beam for probabilistic variations of σ_{cu}, σ_s and dimensions of the section have already been obtained in Example 4.4. The results are

$$M_{rcm} = 67.42 \text{ tm}; \; s_{rc} = 3.99 \text{ tm}$$

From Example 3.7,

$$M_g = 18.75 \text{ tm}; \; M_{qm} = 7.61 \text{ tm}$$

Using Eq. 4.15, the value of p_c of the beam is computed and found to be 4.87×10^{-7}.

Using the parameters of M_{rc} of the critical sections of the beams of Example 4.2 and 4.3 and the values of M_g and M_{qm} at those critical sections from tables4.4 and 4.5, the values of p_c of the above beams for different loading conditions can be calculated. Results are presented in tables 4.6 and 4.7 for two span and three span continuous PSC beams (Examples 4.1 and 4.2) subjected to probabilistic variations of Q, σ_{cu}, σ_s and dimensions of section.

Table 4.6. Parameters of M_{rc} and probability of cracking of the beam in Fig. 3.12 for each loading case for PV of Q, σ_{cu}, σ_s and dimensions of section

Sl. No.	Live Load on Spans	Critical Section	M_g (tm)	M_{qm} (tm)	M_{rcm} (tm)	s_{rc} (tm)	p_{ci}
1	AC & CE	C	18.75	7.62	58.31	3.36	7.02(-6)*
2	AC	B_2	9.38	5.67	52.91	3.03	4.49(-8)
3	CE	D_2	9.38	5.67	52.91	3.03	4.49(-8)

Bounds on p_{os} of the beam : $7.02(-6) \leq p_{cs} \leq 8.01(-6)$

* 7.02(-6) is read as 7.02×10^{-6}.

Table 4.7. Parameters of M_{rc} and probability of cracking of the beam in Fig. 3.13 for each loading case for PV of Q, σ_{cu}, σ_s and dimensions of section

Sl. No.	Live Load on Spans	Critical Section	M_g (tm)	M_{qm} (tm)	M_{rcm} (tm)	s_{rc} (tm)	p_{ci}
1	AC, CE & EG	C or E	6.4	3.12	28.73	1.607	8.32(-8)
2	AC & EG	B_2 or F_2	4.8	3.15	25.24	1.457	3.24(-7)
3	CE	D	1.6	2.34	17.85	1.046	1.60(-7)
4	AC	B_2	4.8	2.85	25.24	1.457	7.92(-8)
5	AC & CE	C	6.4	3.65	28.73	1.607	7.31(-7)
6	EG	F_2	4.8	2.85	25.24	1.457	7.92(-8)
7	CE & EG	E	6.4	3.65	28.73	1.607	7.31(-7)

Bounds on p_{os} of the beam : $7.31(-7) \leq p_{cs} \leq 21.88(-7)$.

It is observed that the probability of cracking of simply supported beam varies from 10^{-9} to 10^{-22} for deterministic load and from 10^{-6} to 10^{-7} for probabilistic load. The probability of cracking of continuous beams varies from 10^{-6} to very small value less than 10^{-24} for deterministic load and from 10^{-5} to 10^{-7} for probabilistic load. The probability of cracking of beams are higher for probabilistic load than deterministic load.

4.3 RELIABILITY ANALYSIS OF PSC BEAMS AT LIMIT STATE OF DEFLECTION

4.3.1 Introduction

Under working loads PSC beams do not crack usually. Since prestressed concrete is assumed as a homogeneous elastic body which obeys quite closely the ordinary laws of flexure, the deflections can be computed by methods available in elementary strength of materials (58). Deflections due to prestress can be computed by considering the concrete as a free body separated from the tendons which are replaced by a system of forces. Total deflection is calculated by superposing deflections due to load and prestress. Codes specify that deflections must not exceed a certain fraction of span when

(i) prestress and dead load are acting and

(ii) prestress, dead load and live load are acting.

Hence the beam is said to become unserviceable and to have reached serviceability limit state caused due to excessive deflection if

the deflection, u, caused by external load exceeds the allowable deflection specified by the code. If p_d is defined as the probability of the structure becoming unserviceable due to excessive deflection constraint, then

$$p_d = P(\ u > \text{allowable deflection}) \tag{4.16a}$$

Allowable deflection is specified as the fraction of span, say $\psi\ell$ where ψ represents the fraction. Now Eq. 4.16a can be rewritten as

$$p_d = P(\ \frac{u}{\ell} > \psi\ ) \tag{4.16b}$$

Denoting $v = \frac{u}{\ell}$, the Eq. 4.16b becomes

$$p_d = P(v > \psi\ ) \tag{4.16c}$$

The deflection of a prestressed concrete beam is a function of several random variables namely, properties of materials, load and geometric properties of the section. Hence the deflection is a random variable. Since deflection is a random variable, v is also random variable whose distribution and parameters are best obtained by Monte Carlo method.

4.3.2 Equations for Determination of Deflection in PSC Beams

It is assumed that cables are fully bonded and the cable of the beam passes through the centroid of the sections at ends of the beam.

The downward deflection, u_d, of a PSC beam under dead load, live load and prestress can be written as

$$u_d = u_g + u_q - u_p \tag{4.17}$$

where u_g, u_q and u_p are deflections due to dead load, live load and prestress respectively. For parabolic cable profile, the equivalent uniformly distributedload, w_p, due to prestress is given by

$$w_p = \frac{8P\,y_c}{\ell^2} \tag{4.18}$$

where P is the total prestressing force in steel, y_c is the central ordinate of the parabolic cable profile and ℓ is the effective span. For uniformly distributed dead load and live load, Eq. 4.17 can be written as

$$u_d = \frac{\ell^4}{E_c I}\left(c_g\,w_g + c_q \bar{w}_q - c_p w_p\right) \tag{4.19}$$

where I is the moment of inertia of the transformed area of cross section and c_g, c_q and c_p are constants which are either known or can be calculated for given loading and the position where the value of deflection is required. Taking the value of effective tensile stress in prestress steel equal to 0.6 σ_s and using Eq. 4.11, the final equation for v_d is obtained as

$$v_d = \left(\frac{u_d}{\ell}\right) = \frac{\ell^3}{18000\,\sqrt{\sigma_{cu}}\,I}\left(c_g\,w_g + c_q w_q - \frac{4.8\,c_p \sigma_s\,A_{ts} y_c}{\ell^2}\right) \tag{4.20}$$

where v_d is the ratio of downward deflection to the effective span of the beam.

The upward deflection, u_u, of a PSC beam under prestress and dead load is

$$u_u = u_p - u_g \tag{4.21}$$

Under dead load conditions, the bottom fibre of the beam is under compressive stress which may be present for considerable period when the live load is not acting. Under this substained stress, the beam has a tendency to deflect upwards due to creep. The I.S. Code (46) gives the following expression for this deflection

$$u_{cc} = \frac{L_u \ \Delta s}{8h} \tag{4.22}$$

where,

u_{cc} = deflection due to creep

L_u = unsupported length of beam

Δs = differential shortening between top and bottom surfaces of the beam due to creep

Assuming creep coefficient of concrete is equal to 2.5 and stresses in concrete at extreme fibres of the beam are equal to allowable values 0.5 σ_{cu} and zero, the Eq. 4.22 reduces to

$$u_{cc} = \frac{L_u \ \sigma_{cu}}{64h \ E_c} \tag{4.23}$$

Therefore final equation for v_u, similar to Eq. 4.20, can be written as

$$v_u = \left(\frac{u_u}{\ell}\right) = \frac{\ell^3}{18000 \ \sqrt{\sigma_{cu}} \ I} \left(\frac{4.8 \ c_p \sigma_s A_{ts} y_c}{\ell^2} - c_g \ w_g \right)$$

$$+ \frac{L_u \sqrt{\sigma_{cu}}}{1152000h} \tag{4.24a}$$

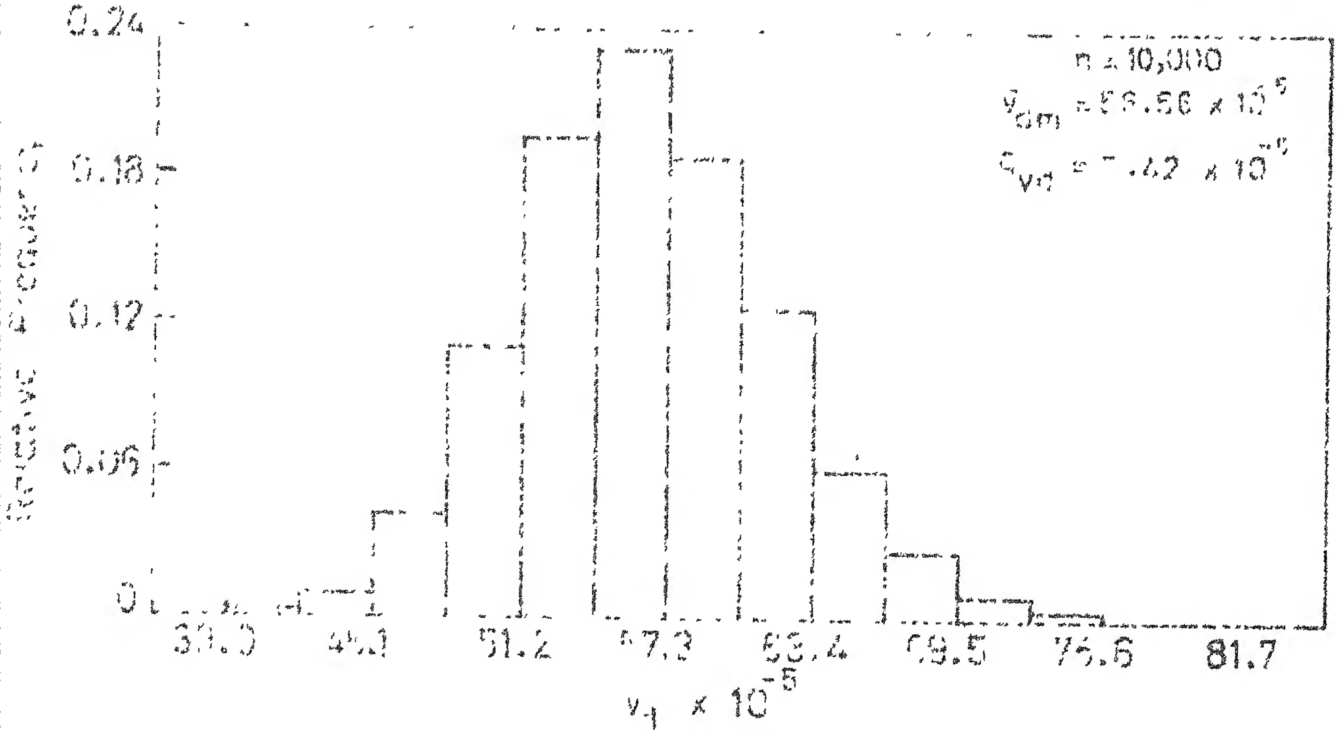
where v_u is the ratio of total upward deflection to the effective span of the beam. Since the moment of inertia is a function of geometric properties of the section, Eqs. 4.20 and 4.24a are functions of strengths of materials, load and geometric properties of the section.

If creep of concrete is not considered, the prestress in steel at transfer should be used instead of effective prestress in determining the deflection due to prestress. Using the maximum value of stress in steel at transfer equal to 0.8 σ_s(46), v_u is expressed as

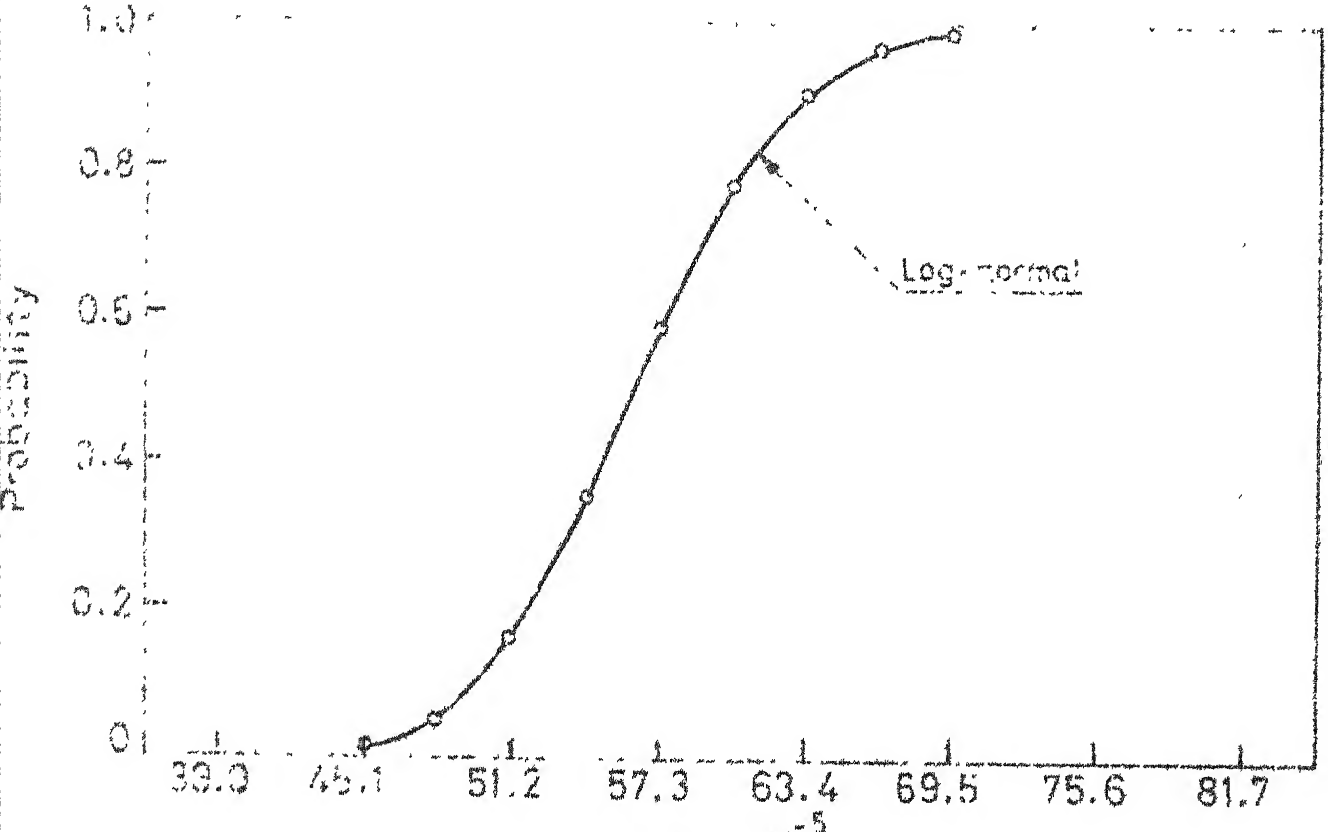
$$v_u = \frac{\ell^3}{18000 \sqrt{\sigma_{cu}}\, I} \left(\frac{6.4\ c_p \sigma_s\ A_{ts} y_c}{\ell^2} - c_g w_g \right) \qquad (4.24b)$$

4.3.3 Probability Distributions of v_d and v_u for Probabilistic Variations of σ_{cu} and σ_s .

The probabilistic variations of σ_{cu} and σ_s are first considered in the equations for determination of v_d and v_u and the reliability analysis at limit state of deflection is first presented for the same. σ_{cu} and σ_s are distributed as N(422.8,56) kg/cm^2 and N(15680,488) kg/cm^2 respectively. The beam of Example 3.1 (Fig. 3.11) is considered. The critical section is midspan of the beam where maximum deflection occurs. The values of c_g, c_q and c_p are same and equal to 0.01302. Using Eqs. 4.20 and distributions and parameters of σ_{cu} and σ_s, 10000 samples are generated for v_d by Monte Carlo method. The histogram and cumulative distribution of the generated data are shown in Fig. 4.4. Lognormal distribution



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG.4.4 VARIATION OF v_d FOR PV OF σ_{cu} AND σ_s

is found to satisfy the generated data at one percent level of significance. Similarly samples have been generated for v_u using Eq. 4.24a and the same lognormal distribution has been observed for this data also. Hence lognormal distribution is adopted for random variables v_d and v_u for probabilistic variations of σ_{cu} and σ_s.

4.3.4 Computation of p_d

The probability of the structure becoming unserviceable due to excessive deflection constraint is given by Eq. 4.16c. I.S. Code (46) specifies values of ψ for v_d and v_u equal to 0.002 and 0.0033 respectively. Therefore Eq. 4.16c for limit state of downward deflection becomes

$$p_{dd} = P(v_d > 0.002) \tag{4.25}$$

and for limit state of upward deflection

$$p_{du} = P\ (v_u > 0.0033) \tag{4.26}$$

where,

p_{dd} = the probability of the structure becoming unserviceable due to excessive downward deflection constraint

p_{du} = the probability of the structure becoming unserviceable due to excessive upward deflection constraint.

For lognormally distributed v_d, the value of p_{dd} can be computed from the following equation

$$p_{dd} = 1 - \phi\ [\frac{\log\ (\frac{0.002}{v_{dm}})}{s_{vd}}] \tag{4.27}$$

where v_{dm} and s_{vd} are the parameters of v_d. Similarly for lognormally distributed v_u, the value of p_{du} can be calculated from the equation

$$p_{du} = 1 - \phi\left[\frac{\log\left(\frac{0.0033}{v_{um}}\right)}{s_{vu}}\right] \tag{4.28}$$

where v_{um} and s_{vu} are the parameters of v_u. The procedure is illustrated with an example.

Example 4.7

The simply supported PSC beam of Example 4.1 (Fig. 3.11) is subjected to probabilistic variations of strengths of materials. Other particulars of the beam remain same as in Example 4.1.

σ_{cu} and σ_s are distributed as N(422.8, 56) kg/cm^2 and N(15680, 488) kg/cm^2 respectively. Using Monte Carlo technique 10000 samples are generated for v_d and v_u at midspan of the beam, given by Eqs. 4.20, 4.24a and 4.24b. The results are given below.

$\bar{v}_{dm} = 56.56 \times 10^{-5}$; $\bar{s}_{vd} = 5.42 \times 10^{-5}$

$\bar{v}_{um} = 45.38 \times 10^{-5}$; $\bar{s}_{um} = 5.00 \times 10^{-5}$ (By Eq. 4.24a)

$\bar{v}_{um} = 86.24 \times 10^{-5}$; $\bar{s}_{um} = 7.85 \times 10^{-5}$ (By Eq. 4.24b)

using similar equations 2.28 and 2.29, the parameters of v_d and v_u can be computed. Their values are given below.

$v_{dm} = 56.30 \times 10^{-5}$; $s_{vd} = 0.0956$

$v_{um} = 45.11 \times 10^{-5}$; $s_{vu} = 0.11$ (For Eq. 4.24a)

$v_{um} = 85.88 \times 10^{-5}$; $s_{vu} = 0.091$ (For Eq. 4.24b)

using Eq. 4.27, the value of p_{dd} is calculated as

$$p_{dd} = 1 - \phi\left[\frac{\log\left(\frac{0.002}{56.3 \times 10^{-5}}\right)}{0.0956}\right] \simeq 0 \quad \text{(i.e. less than } 10^{-24}\text{)}$$

If deflection due to creep of concrete is considered, the value of p_{du}, using Eq. 4.28, is

$$p_{du} = 1 - \phi\left[\frac{\log\left(\frac{0.0033}{45.11 \times 10^{-5}}\right)}{0.11}\right] \simeq 0$$

If deflection due to creep of concrete is not considered, then

$$p_{du} = 1 - \phi\left[\frac{\log\left(\frac{0.0033}{85.88 \times 10^{-5}}\right)}{0.091}\right] \simeq 0$$

Hence the probability of the beam becoming unserviceable due to excessive deflection constraints is zero.

4.3.5 Reliability Analysis of Continuous PSC Beams at Limit State of Deflection

The reliability analysis of continuous PSC beams at limit state of deflection is carried out in the following steps.

(i) Determine the position of maximum deflection for each loading condition.

(ii) using Monte Carlo technique, obtain parameters of v_d and v_u of each critical section.

(iii) Compute p_{dd} and p_{du} for each loading condition using Eqs. 4.27 and 4.28.

(iv) Using Eq. 4.14, calculate bounds on p_{ds} which defines the probability of the beam becoming unserviceable for all loading conditions due to excessive deflection constraint.

The procedure is illustrated with examples.

Example 4.8

The two span PSC beam of Example 4.2 (Fig. 3.12) is considered. The beam is subjected to probabilistic variations of σ_{cu} and σ_s. Other particulars of the beam remain same as in Example 4.2.

σ_{cu} and σ_s are distributed as N(422.8, 56) kg/cm^2 and N(15680, 488) kg/cm^2 respectively. Critical sections where maximum deflection occur under different load conditions are given in table 4.8. The values of constants c_g, c_q and c_p are also given in the same table. Using Monte Carlo method 10000 samples are generated for v_d and v_u of each critical section and the mean value and standard deviation of v_d and v_u are obtained. The results for the critical section X_1(0.422 ℓ from the left support) for load condition (i) are given below.

Load condition (i) : Live load on both spans

Critical section is X_1. The mean value and standard deviation of v_d and v_u are

$$\bar{v}_{dm} = 24.81 \times 10^{-5} \; ; \; \bar{s}_{vd} = 2.38 \times 10^{-5}$$

$$\bar{v}_{um} = 20.60 \times 10^{-5} \; ; \; \bar{s}_{vu} = 2.22 \times 10^{-5} \quad \text{(For Eq. 4.24a)}$$

$$\bar{v}_{um} = 38.46 \times 10^{-5} \; ; \; \bar{s}_{vu} = 3.49 \times 10^{-5} \quad \text{(For Eq. 4.24b)}$$

Table 4.8. Critical sections and values of c_g, c_q and c_p for two span continuous beam

Sl. No.	Live Load on Spans	Critical Section	Distance of Critical Section from End A	c_g	c_q	c_p
1	AC & CE	X_1	0.422 ℓ	0.00542	0.00542	0.00542
2	AC	X_2	0.483 ℓ	0.00527	0.00913	0.00527
3	CE	X_3	1.517 ℓ	0.00527	0.00913	0.00527

Table 4.9. Values of p_{dd} and p_{du} of the beam in Fig. 3.12 for PV of σ_{cu} and σ_s

Sl. No.	Live Load on Spans	Limit State of Downward Deflection			Critical Section	Limit State of Upward Deflection		
		$v_{dm} \times 10^{-5}$	s_{vd}	p_{dd}		$v_{um} \times 10^{-5}$	s_{vu}	p_{du}
1	AC & CE	24.7	0.0957	0	X_1	20.48 *38.30	0.108 0.091	0 0
2	AC	66.95	0.0722	0	X_2			
3	CE	66.95	0.0722	0	X_3			

The value of p_{ds} is equal to zero.

* For Eq. 4.24b (without considering creep of concrete).

Coefficient of variation of $\bar{v}_{dm} = \delta_{vd} = \frac{2.38}{24.81} = 0.096$

Using similar Eqs. 2.21 and 2.22,

$$s_{vd} = \sqrt{\log (0.096^2 + 1)} = 0.0957$$

$$v_{dm} = 24.81 \times 10^{-5} \exp \left(- \frac{1}{2} 0.0957^2\right)$$

$$= 24.70 \times 10^{-5}$$

Similarly parameters of v_u are calculated and given in table 4.9. Using Eqs. 4.27 and 4.28, the values of p_{dd} and p_{du} are calculated as illustrated in the previous example and found to be zero.

Similarly the values of parameters of v_d and v_u for other load conditions are obtained and shown in table 4.9. The values of p_{dd} for other load conditions are calculated in a similar manner and given in the same table. In every loading case, the value of p_{dd} is found to be zero and hence the probability of the beam becoming unserviceable due to excessive deflection constraint for all load conditions is zero.

Example 4.9

The three span continuous PSC beam of Example 4.3 (Fig. 3.13) is considered. The beam is subjected to probabilistic variations of σ_{cu} and σ_s. Other particulars of the beam remain same as in Example 4.3.

Distributions and parameters of σ_{cu} and σ_s remain same as in Example 4.3. The critical sections for limit states of deflection

for each loading case is given in table 4.10. The values of c_g, c_q and c_p are also given in the same table. Using Monte Carlo method 10000 samples are generated for v_d and v_u of each critical section. Parameters of v_d and v_u are calculated from the mean values and standard deviations of v_d and v_u obtained from the Monte Carlo method. Parameters of v_d and v_u for all critical sections are given in table 4.11 Using Eqs. 4.27 and 4.28, the values of p_{dd} and p_{du} are computed as illustrated in the previous examples. Table 4.11 shows the values of p_{dd} and p_{du} for each loading case. It is found that values of p_d is zero in every case and so the probability of the beam becoming unserviceable due to excessive deflection constraint for all loading conditions is zero.

4.3.6 Probability Distribution of v_d for Probabilistic Variations of σ_{cu}, σ_s and Q

The probabilistic variations of σ_{cu}, σ_s and Q are considered in the equation 4.20. For reliability analysis, the probability distribution of v_d is required. To study the probability distribution of v_d, the beam of Example 4.1 is considered. σ_{cu}, σ_s and Q are distributed as N(422.8, 56) kg/cm^2, N(15680, 488) kg/cm^2 and LN(608.85 kg/m, 0.368) respectively. The maximum deflection occurs at midspan of the beam. The values of c_g, c_q and c_p are same and equal to 0.01302. Using Eq. 4.20 and distributions and parameters of σ_{cu}, σ_s and Q 10000 samples are generated for v_d by Monte Carlo method. The histogram and cumulative distribution of the generated data are shown in Fig. 4.5. The distribution is highly positively

Table 4.10. Critical section and values of c_g, c_q and c_p for three span continuous beam

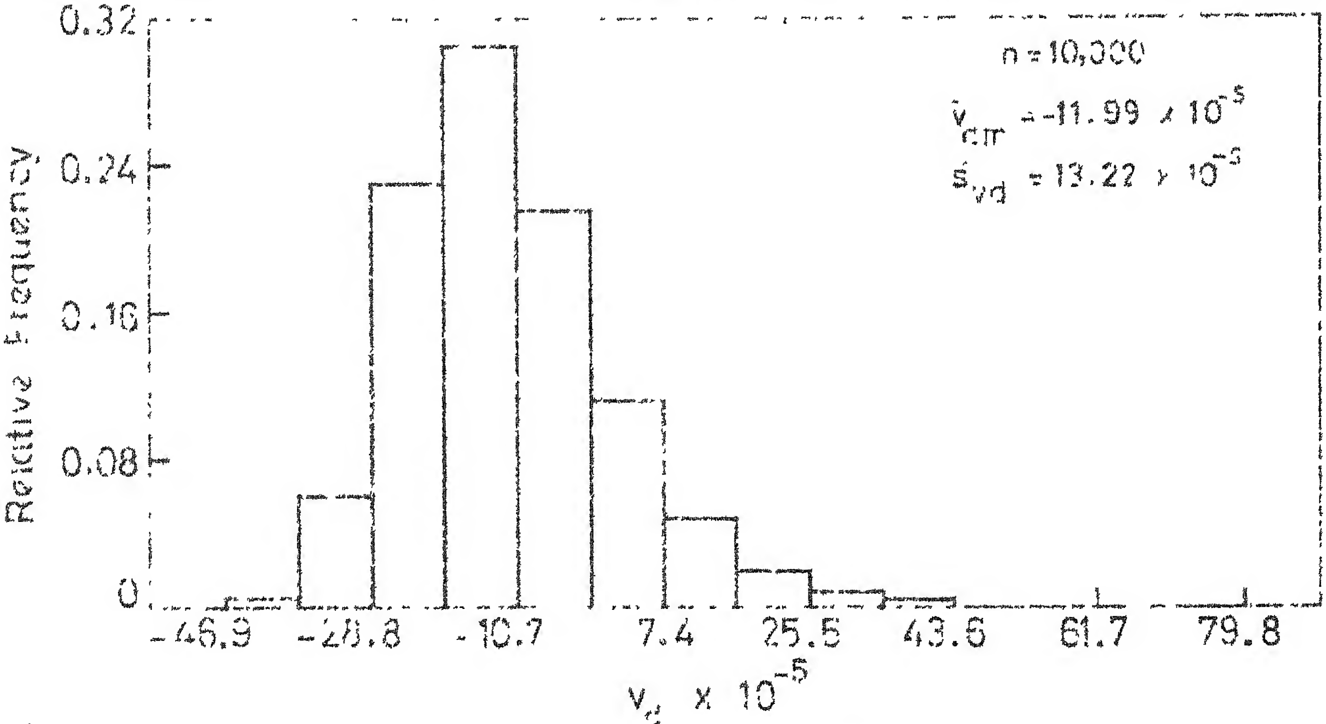
Sl. No.	Live Load on Spans	Critical Section	Distance of Critical Section from End A	c_g	c_q	c_p
1	AC,CE & EG	X_1	0.444 ℓ	0.00693	0.00693	0.00693
2	AC & EG	X_2	0.496 ℓ	0.00677	0.00990	0.00677
3	CE	X_3	1.5 ℓ	0.00052	0.00677	0.00052
4	AC	X_4	0.416 ℓ	0.00684	0.00887	0.00684
5	AC & CE	X_5	0.483 ℓ	0.00685	0.00587	0.00685
6	EG	X_6	2.584 ℓ	0.00684	0.00887	0.00684
7	CE & EG	X_7	2.517 ℓ	0.00685	0.00587	0.00685

Table 4.11. Values of p_{dd} and p_{du} of the beam in Fig. 3.13 for PV of σ_{cu} and σ_s

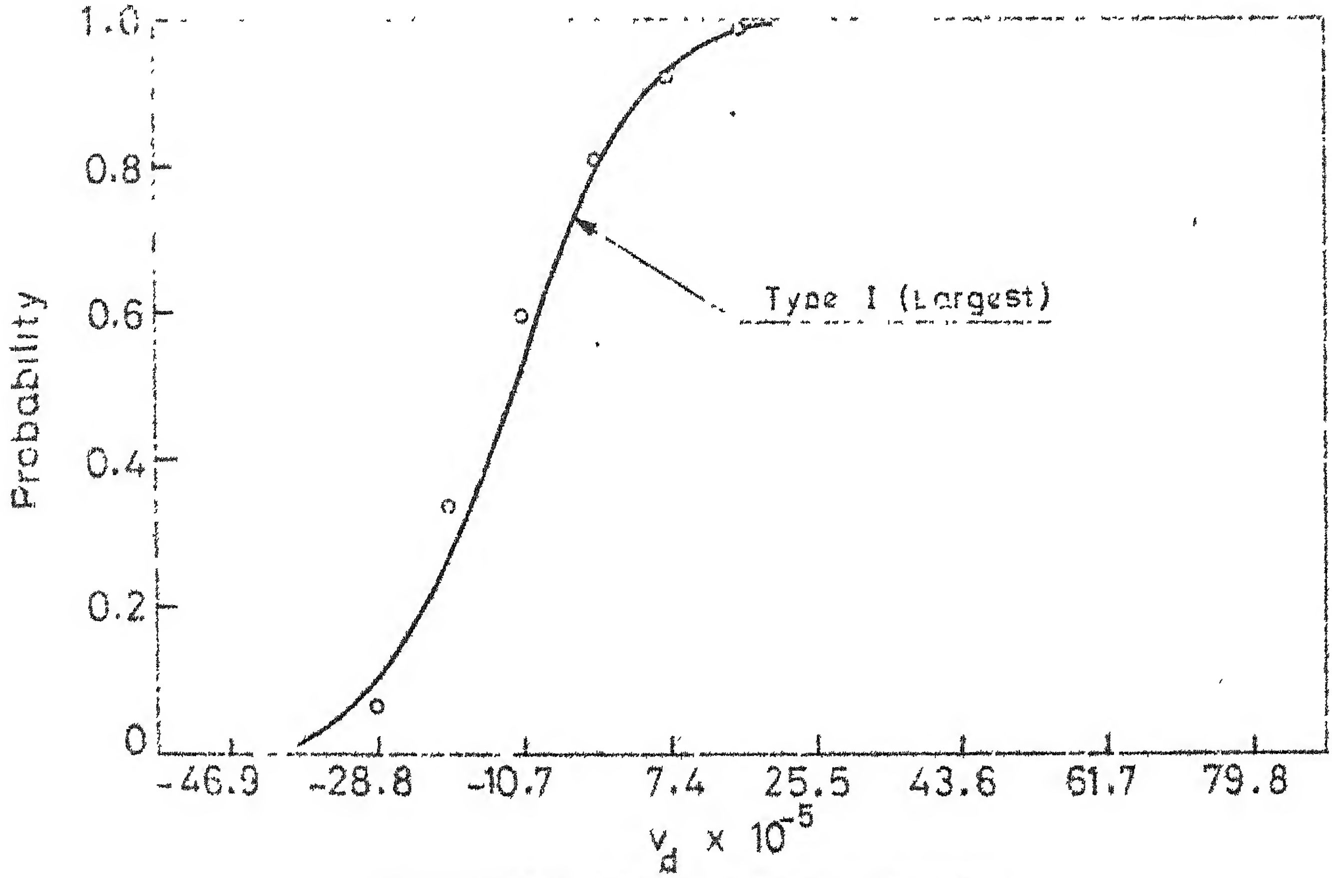
Sl. No.	Live Load on Spans	Limit State of Downward Deflection			Critical Section	Limit State of Upward Deflection		
		$v_{dm} \times 10^{-5}$	s_{vd}	p_d		$v_{um} \times 10^{-5}$	s_{vu}	p_{du}
1	AC,CE & EG	31.62	0.0960	0	X_1	29.83 *51.89	0.0996 0.0872	0 0
2	AC & EG	58.05	0.0763	0	X_2			
3	CE	65.55	0.0686	0	X_3			
4	AC	48.90	0.081	0	X_4			
5	AC & CE	22.29	0.116	0	X_5			
6	EG	48.90	0.081	0	X_6			
7	CE & EG	22.29	0.116	0	X_7			

The value of p_{ds} is equal to zero.

*For Eq. 4.24b (without considering creep of concrete).



(a) HISTOGRAM



(b) CUMULATIVE DISTRIBUTION

FIG. 4.5 VARIATION OF v_d FOR PV OF σ_{cu}', σ_s AND Q

skewed. The random variable v_d has taken positive and negative values. This is true in reality also as prestressed concrete beam can deflect upwards and downwards depending upon the magnitude of live load on the beam. The standard distribution which has these characteristics i.e. high positive skewness and positive and negative values for random variable, is Type I (largest) extremal distribution. This distribution does not satisfy chi square test because of large values of chi square on the lower tail even though the distribution fits the data well on the remaining portion. However as this is the most conceivable distribution for the deflection behaviour of prestressed concrete beams, Type I (largest) extremal distribution is assumed for v_d for probabilistic variation of σ_{cu}, σ_s and Q.

For the Type I (largest) extremal distribution, the cumulative probability of v_d is given by (48)

$$F_{v_d}(v_d) = \exp\left[-\exp\{-s_{vd}(v_d - v_{dm})\}\right] \qquad -\infty \le v_d \le \infty \tag{4.29}$$

in which v_{dm} and s_{vd} are parameters of v_d. The above parameters are evaluated from the following equations (48).

$$v_{dm} = \bar{v}_{dm} - \frac{0.577}{s_{vd}} \tag{4.30}$$

$$s_{vd} = \frac{\pi}{\sqrt{6}\,\bar{s}_{vd}} \tag{4.31}$$

The format for the probability of the structure becoming unserviceable due to excessive downward deflection constraint is given

by Eq. 4.25. Hence if v_d is assumed to follow Type I extremal (largest) distribution, the value of p_{dd}, using Eq. 4.29, is obtained from the following equation.

$$p_{dd} = 1 - \exp\ [-\exp\ \{ -s_{vd}\ (0.002 - v_{dm})\}] \tag{4.32}$$

The method of reliability analysis at limit state of deflection for probabilistic variation of σ_{cu}, σ_s and Q is illustrated with an example.

Example 4.10

The simply supported beam of Example 4.1 (Fig. 3.11) is analysed at limit state of deflection and for probabilistic variations of σ_{cu}, σ_s and Q.

Using Monte Carlo method, the values of $\bar{v}_{dm}$ and $\bar{s}_{vd}$ have already been obtained for this beam in the previous section. From Fig. 4.5, the values of $\bar{v}_{dm}$ and $\bar{s}_{vd}$ are -11.99×10^{-5} and 13.22×10^{-5} respectively. Using Eqs. 4.30 and 4.31, the parameters of v_d are calculated as

$$s_{vd} = \frac{\pi \times 10^5}{\sqrt{6}\ 13.22} = 0.0965 \times 10^5$$

$$v_{dm} = (-11.99 - \frac{0.577}{0.0965})10^{-5}$$

$$= -17.97 \times 10^{-5}$$

Substituting the above values of s_{vd} and v_{dm} in Eq. 4.32, the value of p_{dd} is obtained as 7.27×10^{-10}. The value of p_{du} remains same as calculated in Example 4.9.

The reliability analysis of continuous beams at limit state of deflection and for probabilistic variations of σ_{cu}, σ_s and Q is same as explained in article 4.3.5 except that probabilistic variation of Q is also to be taken into account in determining parameters of v_d and Eq. 4.32 must be used to compute p_{dd}. The results of the reliability analysis at limit state of deflection and for probabilistic variation of σ_{cu}, σ_s and Q for continuous PSC beams of Examples 4.2 and 4.3 are given in tables 4.12 and 4.13 respectively.

4.3.7 Probability Distributions of v_d and v_u for Probabilistic Variations of σ_{cu} , σ_s , Q and Dimensions of Section

In the previous sections probabilistic variations of strengths of materials and live load only have been considered in determining the probability distributions of v_d and v_u. As it is known from chapter 2 that dimensions of section of a beam have also random variations which are now taken into account in the equations of v_d and v_u. The beam of Example 4.1 (Fig. 3.11) is considered. The mean values, standard deviations and distributions of σ_{cu}, σ_s , Q, b_t, b', t_t, t_b, d, b_b, h and D_s are available in the Example 4.3. Using them and prediction Eqs. 4.20 and 4.24a, 10000 samples are generated for v_d and v_u by Monte Carlo method. The histograms and cumulative distributions of v_d and v_u are shown in Figs. 4.6 and 4.7 respectively. It is observed from the Fig. 4.6, that the distribution of the generated data for v_d is highly positively skewed and the random variable v_d takes positive and negative values. Even

Table 4.12. Values of p_{dd} and p_{du} of the beam in Fig. 3.12 for PV of σ_{cu}, σ_s and Q

Sl. No.	Live Load on Spans	Limit State of Downward Deflection			Critical Section	Limit State of Upward Deflection		
		$v_{dm} \times 10^{-5}$	$s_{vd} \times 10^{5}$	p_{dd}		$v_{um} \times 10^{-5}$	s_{uv}	p_{du}
1	AC and CE	-8.18	0.221	0	X_1	20.48 *38.30	0.108 0.091	0 0
2	AC	3.51	0.123	2.88(-11)	X_2			
3	CE	3.51	0.123	2.88(-11)	X_3			

Bounds on p_{ds} of the beam for

limit state of downward deflection : $2.88(-11) \leq p_{ds} \leq 5.76(-11)$

* For Eq. 4.24b (without considering creep of concrete).

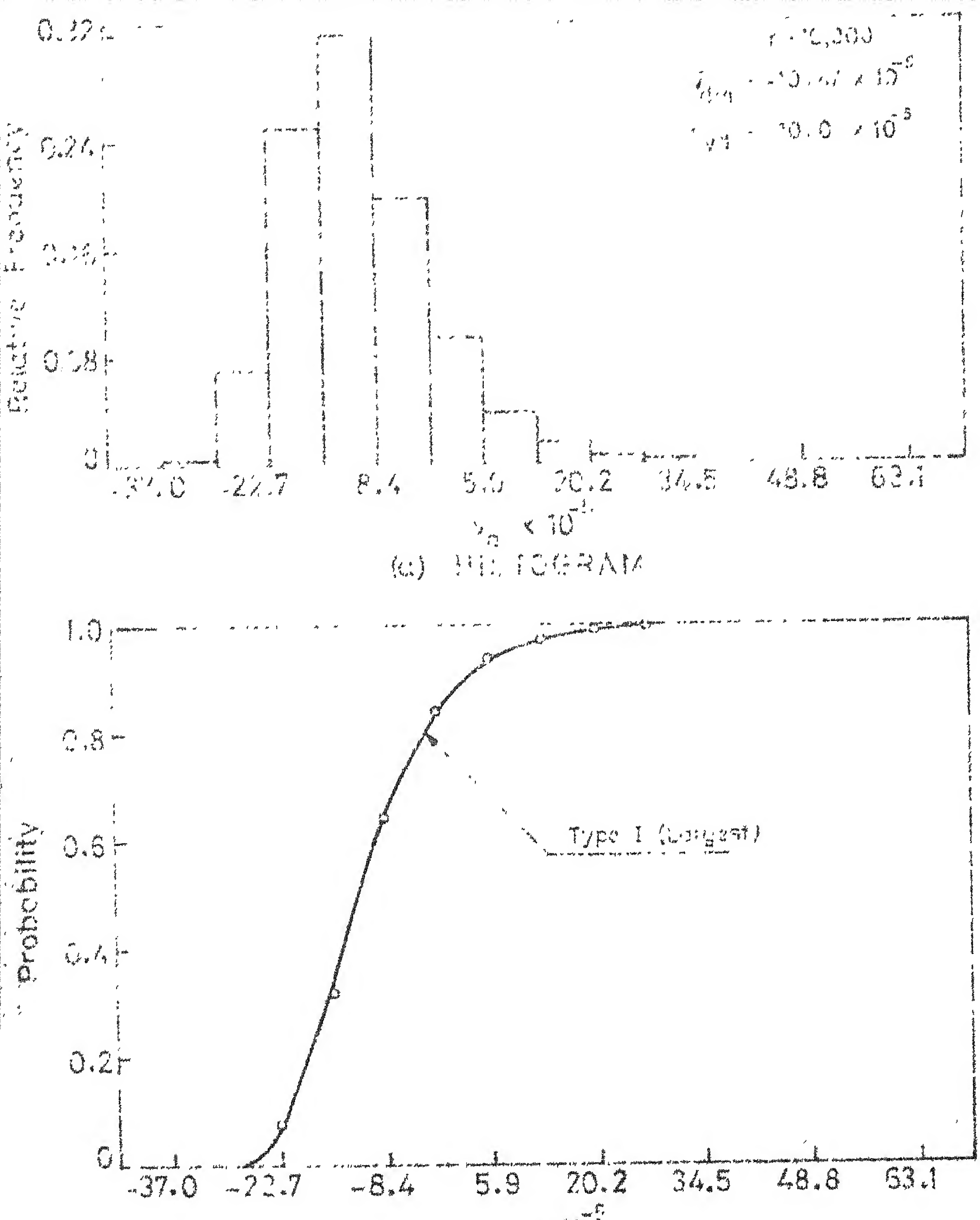
Table 4.13. Values of p_{dd} and p_{du} of the beam in Fig. 3.13 for PV of σ_{cu}, σ_s and Q

Sl. No.	Live Load on Spans	Limit State of Downward Deflection			Critical Section	Limit State of Upward Deflection		
		$v_{dm} \times 10^{-5}$	$s_{vd} \times 10^{5}$	p_{dd}		$v_{um} \times 10^{-5}$	s_{vu}	p_{du}
1	AC, CE & EG	-13.17	0.162	1.0(-15)+	X_1	29.83 *51.89	0.0996 0.0872	0 0
2	AC & EG	- 5.29	0.117	3.4(-11)	X_2			
3	CE	7.60	0.169	2.3(-14)	X_3			
4	AC	- 8.13	0.129	2.08(-12)	X_4			
5	AC & CE	-15.52	0.193	0	X_5			
6	EG	- 8.13	0.129	2.08(-12)	X_6			
7	CE & EG	-15.52	0.193	0	X_7			

Bounds on p_{ds} of the beam at limit state of downward deflection : $3.4(-11) \leq p_{ds} \leq 4.02(-11)$

* For Eq. 2.24a (without considering creep of concrete)

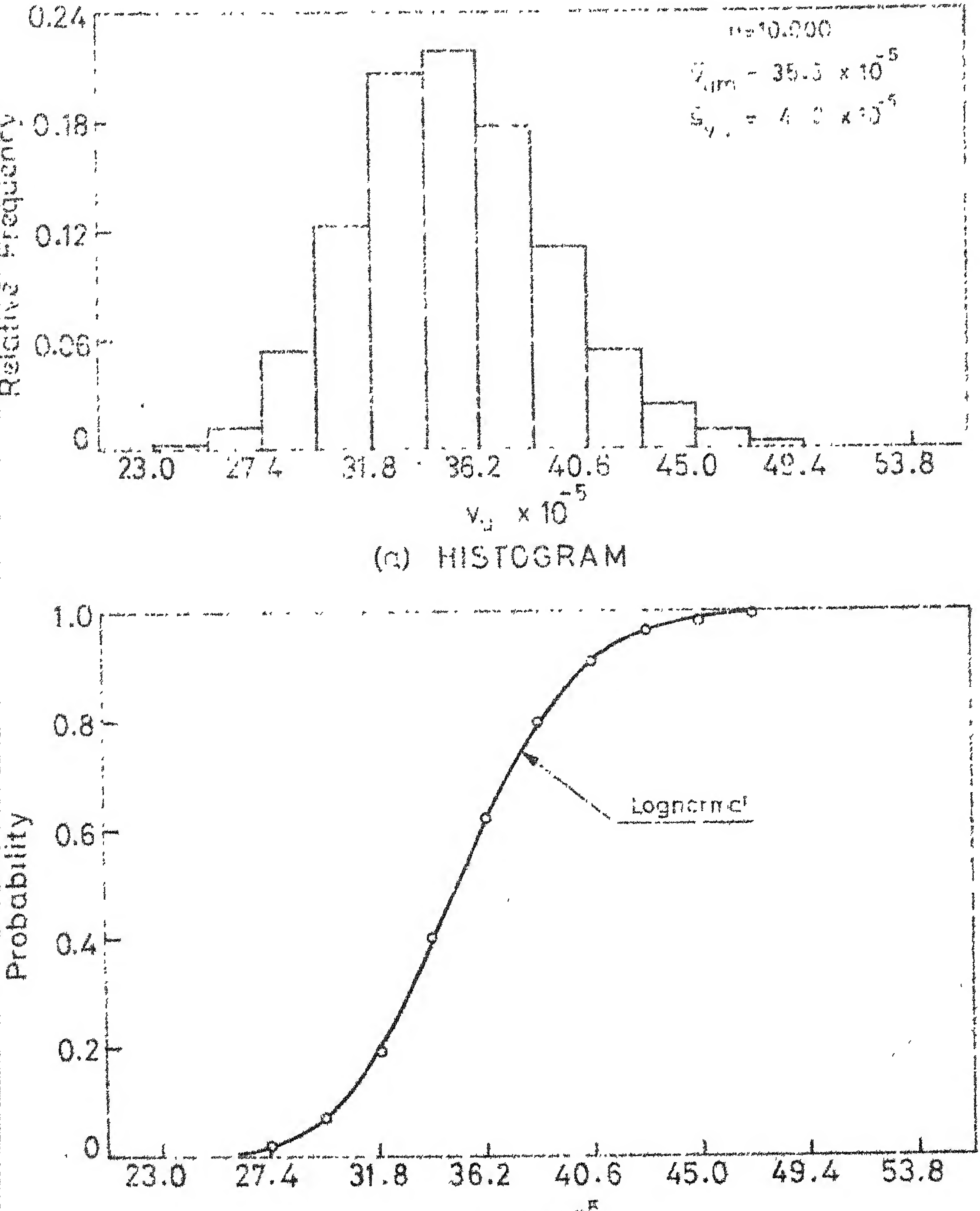
+ 1.0(-15) is read as 1.0×10^{-15}.



(a) HISTOGRAM

(b) CUMULATIVE DISTRIBUTION

FIG.4.6 VARIATION OF v_d FOR PV OF σ_{cu}, σ_s, Q AND DIMENSIONS OF SECTION



(b) CUMULATIVE DISTRIBUTION

FIG.4.7 VARIATION OF v_u FOR PV OF σ_{cu}, σ_s, AND DIMENSIONS OF SECTION

though Type I (largest) extremal distribution does not satisfy chi square test at one percent level of significance, the same distribution is assumed for v_d based on reasons given in the previous article. Estimation of parameters of the above distribution and computation of p_{dd} remain same as explained in the previous article.

Lognormal distribution is found to satisfy the chi square test at one percent level of significance for the generated data shown in Fig. 4.7 for the random variable v_u. Hence lognormal distribution is adopted for v_u for probabilistic variation of strengths of materials, load and geometric properties of the section.

An example is presented to illustrate the method of reliability analysis at limit states of deflection and probabilistic variations of σ_{cu}, σ_s, Q and dimensions of section.

Example 4.11

The simply supported beam of Example 4.1 (Fig. 3.11) is analysed at limit state of deflection with probabilistic variations of σ_{cu}, σ_s, Q and dimensions of section. The live load is distributed as LN(608.85 kg/m, 0.368). Other particulars of the beam remain same as in Example 4.1.

Using Monte Carlo method, the values of $\bar{v}_{dm}$, $\bar{s}_{vd}$, $\bar{v}_{um}$ and $\bar{s}_{vu}$ at the midspan of the beam have already been determined and shown in Figs. 4.6 and 4.7. From the above mentioned figures

$$\bar{v}_{dm} = -10.47 \times 10^{-5} \; ; \; \bar{s}_{vd} = 10.0 \times 10^{-5}$$

$$\bar{v}_{um} = 35.3 \times 10^{-5} \; ; \; \bar{s}_{vu} = 4.01 \times 10^{-5} \qquad \text{(For Eq. 4.24a)}$$

$$\bar{v}_{um} = 66.0 \times 10^{-5} \; ; \; \bar{s}_{vu} = 6.35 \times 10^{-5} \qquad \text{(For Eq. 4.24b)}$$

Assuming Type I (largest) extremal distribution for v_d, the parameters of v_d are calculated by using Eqs. 4.30 and 4.31.

$$v_{dm} = -14.91 \times 10^{-5} \; ; \; s_{vd} = 0.127 \times 10^{5}$$

Using Eq. 4.32, the value of p_{dd} is obtained as 1.81×10^{-12}.

For lognormally distributed v_u, the parameters, calculated using similar Eqs. 2.21 and 2.22, are found to be

$$v_{um} = 35.07 \times 10^{-5} \; ; \; s_{vu} = 0.113 \qquad \text{(For Eq. 4.24a)}$$

$$v_{um} = 65.69 \times 10^{-5} \; ; \; s_{vu} = 0.096 \qquad \text{(For Eq. 4.24b)}$$

Using Eq. 4.28, the values of p_{du} for both cases, i.e. considering the deflection due to creep of concrete and without considering the deflection due to creep of concrete, are found to be zero.

The reliability analysis of continuous beams at limit state of deflection with probabilistic variations of σ_{cu}, σ_s, Q and dimensions of section remain same as explained in the previous section except that probabilistic variations of dimensions of section are also to be taken into account in determing parameters of v_d and v_u. The results of the reliability analysis of the two span and three span

continuous PSC beams shown in Figs. 3.12 and 3.13 are presented in tables 4.14 and 4.15 respectively.

It is observed that the probability of PSC beams becoming unserviceable due to excessive deflection constraint at transfer of prestress is almost zero. The probability of PSC beams becoming unserviceable due to excessive deflection constraint at design load is almost zero for deterministic load. For probabilistic load, the values of p_{dd} for continuous beams are found to be smaller than the values of p_{dd} for simply supported beams as expected.

Table 4.14. Values of p_{dd} and p_{du} of the beam in Fig. 3.12 for PV of σ_{cu}, σ_s, Q and dimensions of section

Sl. No.	Live Load on Spans	Limit State of Downward Deflection			Critical Section	Limit State of Upward Deflection		
		$v_{dm} \times 10^{-5}$	$s_{vd} \times 10^{5}$	p_{dd}		$v_{um} \times 10^{-5}$	s_{vu}	p_{du}
1	AC & CE	-8.059	0.253	0	X_1	18.84	0.113	0
						34.75+	0.096	0
2	AC	2.245	0.142	6.97(-13)*	X_2			
3	CE	2.245	0.142	6.97(-13)	X_3			

Bounds on p_{ds} of the beam at limit state of downward deflection : $6.97(-13) \leq p_{ds} \leq 13.94(-13)$

+ For Eq. 2.24a (without considering creep of concrete)

* 6.9(-13) is read as 6.9×10^{-13}.

Table 4.15. Values of p_{dd} and p_{du} of the beam in Fig. 3.13 for PV of σ_{cu} , σ_s, Q and dimensions of section

Sl. No.	Live Load on Spans	Limit State of Downward Deflection			Critical Section	Limit State of Upward Deflection		
		$v_{dm} \times 10^{-5}$	$s_{vd} \times 10^{5}$	p_{dd}		$v_{um} \times 10^{-5}$	s_{vu}	p_{du}
1	AC, CE & EG	-12.62	0.181	0	X_1	26.92 47.03+	0.104 0.092	0 0
2	AC & EG	- 5.79	0.132	1.63(-12)*	X_2			
3	CE	12.05	0.198	0	X_3			
4	AC	- 8.14	0.144	9.43(-15)	X_4			
5	AC & CE	-14.68	0.22	0	X_5			
6	CE	- 8.14	0.144	9.43(-15)	X_6			
7	CE & EG	-14.68	0.22	0	X_7			

Bounds on p_{ds} of the beam at limit state of downward deflection : $1.63(-12) \leq p_{ds} \leq 1.65(-12)$

+ For Eq. 4.24b (without considering creep of concrete)

* 1.63(-12) is read as 1.63×10^{-12}.

CHAPTER 5

RELIABILITY ANALYSIS OF PRESTRESSED CONCRETE BEAMS AT TRANSFER OF PRESTRESS

5.1 INTRODUCTION

Different stages of loading are to be considered in the analysis and design of PSC beams. In Chapter 3, reliability analysis of PSC beams at limit state of strength at design load were presented. In this chapter, reliability analysis of PSC beams at limit state of strengthat transfer of prestress is given considering probabilistic variations of :

(i) strengths of materials

(ii) strengths of materials and geometric properties of the section.

The failure of bottom flange under compression of concrete at transfer of prestress occurs if the strain in the bottom flange reaches ε_c. The limit state is reached when the strain in bottom fibre is equal to ε_c or the stress at the bottom flange is equal to the compressive strength of concrete. At this limit state, (i) total compressive force in concrete, C_c, should be equal to the prestressing force at transfer P_t and (ii) the distance of the centroid of the area under compression from centre of gravity of steel must be equal to (M_g/P_t). As in the case of failure under maximum loading, it is assumed that the stress distribution can be represented by a rectangular diagram

(uniform stress) over a depth 'a'. The strain at the extreme fibre in compression is equal to the rupture strain 0.0035 and the uniform stress is equal to 0.68 σ_{cu}. For calculating probability of failure or checking safety, the failure can be said to occur when the resisting capacity of the section is less than the prestressing force. If S_b is the area under compression (shown hatched in Fig. 5.1), then the probability of failure of the section at transfer of prestress, p_t, is given by

$$p_t = P(0.68\ \sigma_{cu}\ S_b < P_t) \tag{5.1}$$

Since S_b is a function of P_t, the above equation can be rewritten as

$$p_t = P(\kappa < 1) \tag{5.2}$$

where

$$\kappa = 0.68\ \sigma_{cu}\ S_b/P_t \tag{5.3}$$

κ represents the ratio of resistance to action.

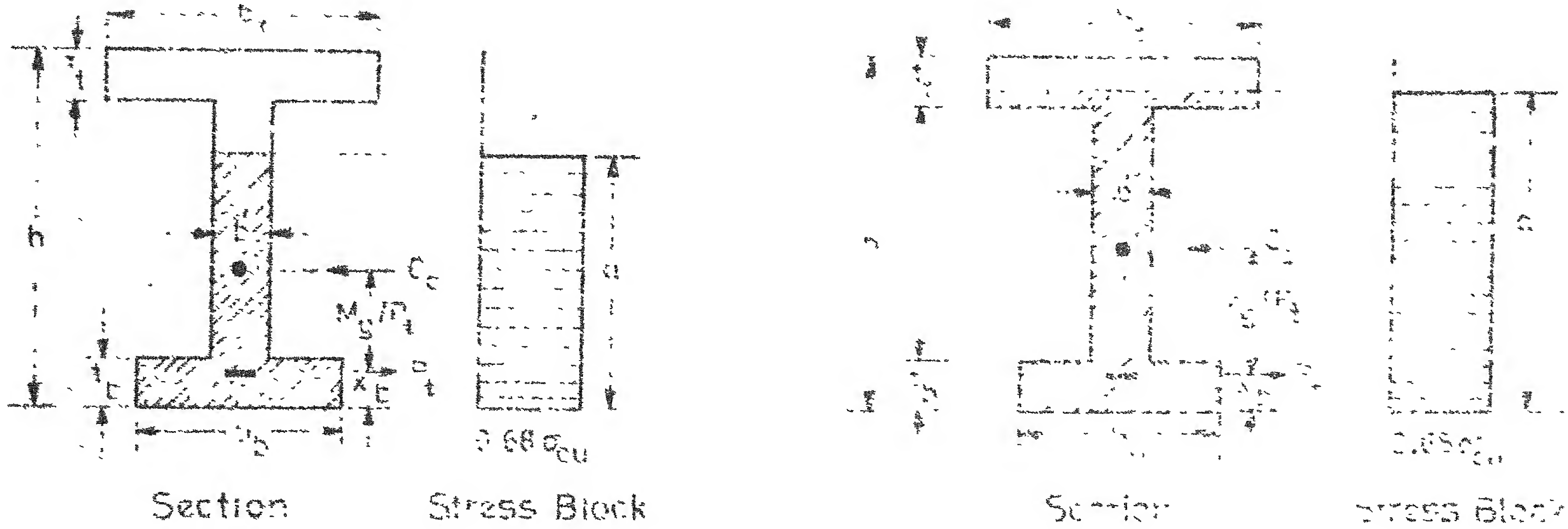
5.2 EQUATIONS FOR DETERMINATION OF κ

The failure of the bottom flange of a flanged section at transfer of prestress may take place with any one of the following events :

E_1 - the event that $a < h - t_t$ (Fig. 5.1a)

E_2 - the event that $a > h - t_t$ (Fig. 5.1b)

Following equations are given to determine κ .



(a) EVENT E_1: $a < (h - t_t)$

(b) EVENT E_2: $a \geq (h - t_t)$

FIG.5.1 STATE OF STRESS AT LIMIT STATE OF STRENGTH OF BOTTOM FLANGE AT TRANSFER OF PRESTRESS

Event E_1 (Fig. 5.1a)

If 'a' is less than $(h - t_t)$, the area of concrete in compression is given by

$$S_b = b' a + t_b (b_b - b') \tag{5.4}$$

where t_b is the thickness of bottom flange. At this limit state, the distance of centroid of S_b from the line of action of P_t must be equal to the shift of line of action of C_c from the line of action of P_t where C_c denotes compressive force in concrete. Using the above condition, the following equation is obtained.

$$S_b = \frac{P_t}{M_g} [b' a(\frac{a}{2} - x_b) - t_b(b_b - b') (x_b - \frac{t_b}{2})] \tag{5.5}$$

where x_b is the distance of centroid of steel from bottom. Solving Eqs. 5.4 and 5.5, the value of 'a' is expressed as

$$a = (b_2 + \sqrt{b_2^2 + 4b_1 b_3})/(2b_1) \tag{5.6}$$

where

$$b_1 = \frac{P_t b'}{2M_g}$$

$$b_2 = b' (1 + x_b \frac{P_t}{M_g})$$

$$b_3 = t_b(b_b - b') [1 + \frac{P_t}{M_g} (x_b - \frac{t_b}{2})]$$

The value of 'a' obtained by Eq. 5.6 is substituted in Eq. 5.4 to get the value of S_b. Knowing S_b, the value of κ can be computed by using Eq. 5.3.

Event E_2 (Fig. 5.1b)

If 'a' is greater than $(h - t_t)$ the equation for the value of 'a' can be similarly derived as for event E_1 and given by

$$a = (c_2 + \sqrt{c_2^2 + 4c_1 c_3})/(2c_1) \tag{5.7}$$

where

$$c_1 = b_t P_t/(2M_g)$$

$$c_2 = b_t \left(1 + x_b \frac{P_t}{M_g}\right)$$

$$c_3 = t_b (b_b - b') \left[1 + \frac{P_t}{M_g} (x_b - 0.5 t_b)\right]$$

$$+ (b_t - b') (t_t - h) \left[1 - \frac{P_t}{2M_g} (h - t_t - 2 x_b)\right]$$

The value of S_b is expressed as

$$S_b = b'a + t_b(b_b - b') + (a - h + t_t) (b_t - b') \tag{5.8}$$

Knowing S_b, the value of κ can be computed by using Eq. 5.3.

There is a possibility that failure may occur at the ends of a PSC beam. The failure referred here is the direct strength failure. Secondary stresses (bursting and spalling) developed in the end block (anchorage zone) as a consequence of transfer of prestressing force at the ends of the beam are not considered in this thesis. Assuming the centroid of steel coincides with the centroid of the section at the supported end of the beam, the value of S_b for rectangular shape $(b_t \times h)$ of the end section can be expressed as

$$S_b = b_t h \tag{5.9a}$$

In case, the centroid of the cable has an eccentricity at ends of the beam, the value of S_b for rectangular section is

$$S_b = 2b_t (h - d) \tag{5.9b}$$

Knowing S_b, κ can be computed by using Eq. 5.3.

The stress in prestress steel is assumed as 0.8 σ_s, the maximum value specified by I.S. Code (46). Hence

$$P_t = 0.8\ \sigma_s\ A_{ts} \tag{5.10}$$

It can be seen in all cases, κ is a function of σ_s , σ_{cu} , M_g, A_{ts} and dimensions of section. Monte Carlo method has been used to determine probability distribution and parameters of κ .

5.3 PROBABILITY DISTRIBUTION OF κ FOR PROBABILISTIC VARIATIONS OF STRENGTHS OF MATERIALS

Probabilistic variations of σ_{cu} and σ_s are first considered in governing Eqs. 5.4 to 5.10 used to determine κ. The prestressed concrete section shown in Fig. 3.11 is considered. σ_{cu} and σ_s are distributed as N(422.8, 56) kg/cm^2 and N(15680, 488) kg/cm^2 respectively. Using Monte Carlo method, 10000 samples are generated for κ of the section using Eqs. 5.4 to 5.8. The histogram and cumulative distribution of the generated data are shown in Fig. 5.2. The normal distribution is found to satisfy the chi square test at one percent level of significance. Hence normal distribution is adopted for κ for probabilistic variations of σ_{cu} and σ_s.

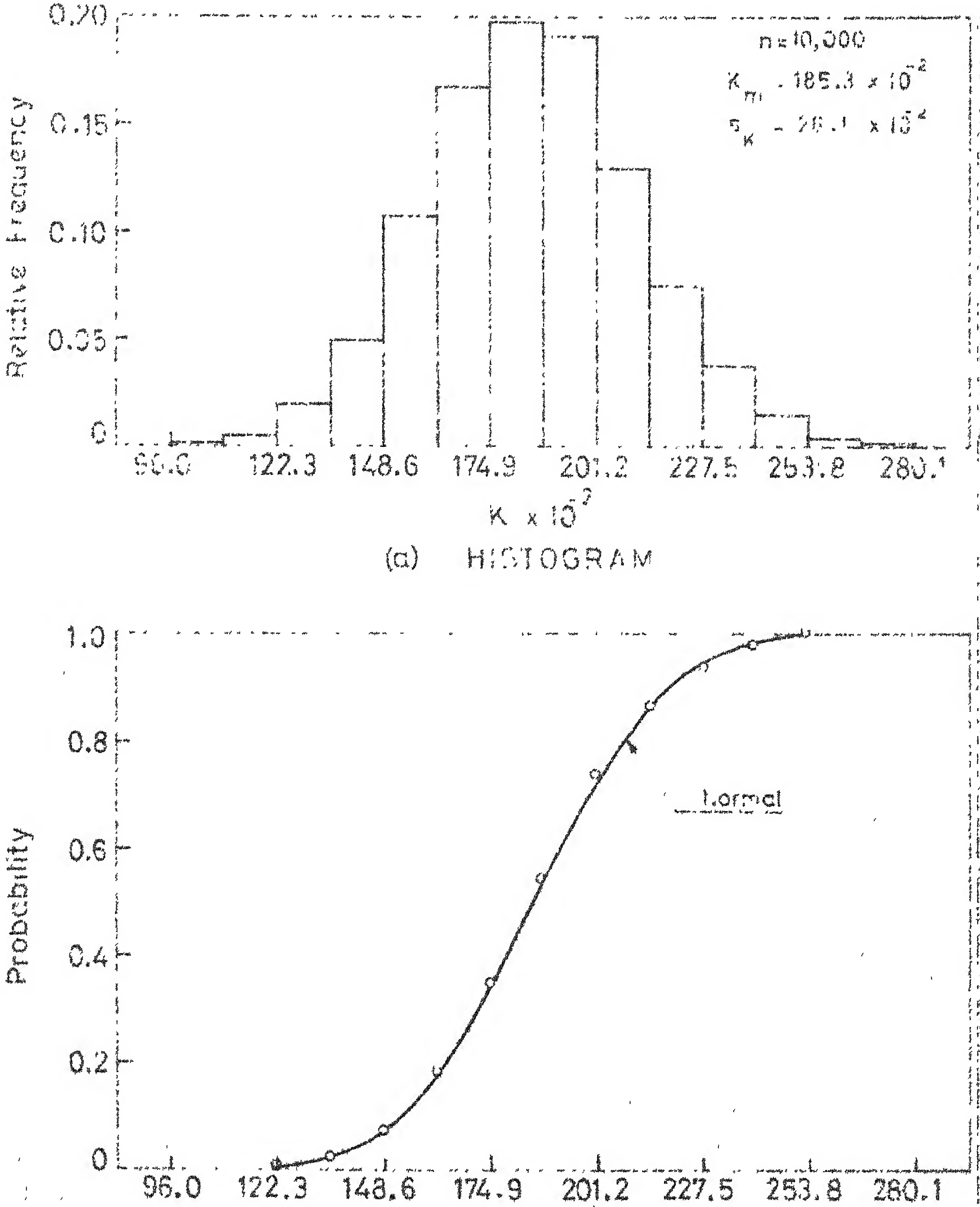


FIG. 5.2 VARIATION OF K FOR PV OF σ_{cu} AND σ_s

5.4 COMPUTATION OF PROBABILITY OF FAILURE AT TRANSFER OF PRESTRESS

The probability of failure of a section at limit state of strength at transfer of prestress is given by Eq. 5.2. The value of p_t for normally distributed κ is given by

$$p_t = P(\kappa < 1) = \phi\left(\frac{1 - \kappa_m}{s_\kappa}\right) \tag{5.11}$$

where κ_m and s_κ are the parameters mean and standard deviation of κ .

5.5 RELIABILITY ANALYSIS OF A SIMPLY SUPPORTED PSC BEAM AT STRENGTH LIMIT STATE AT TRANSFER OF PRESTRESS

The following steps are involved in the reliability analysis of a simply supported PSC beam at limit state of strength at transfer of prestress.

(i) For the given beam, determine the critical section where maximum compressive stress occurs.

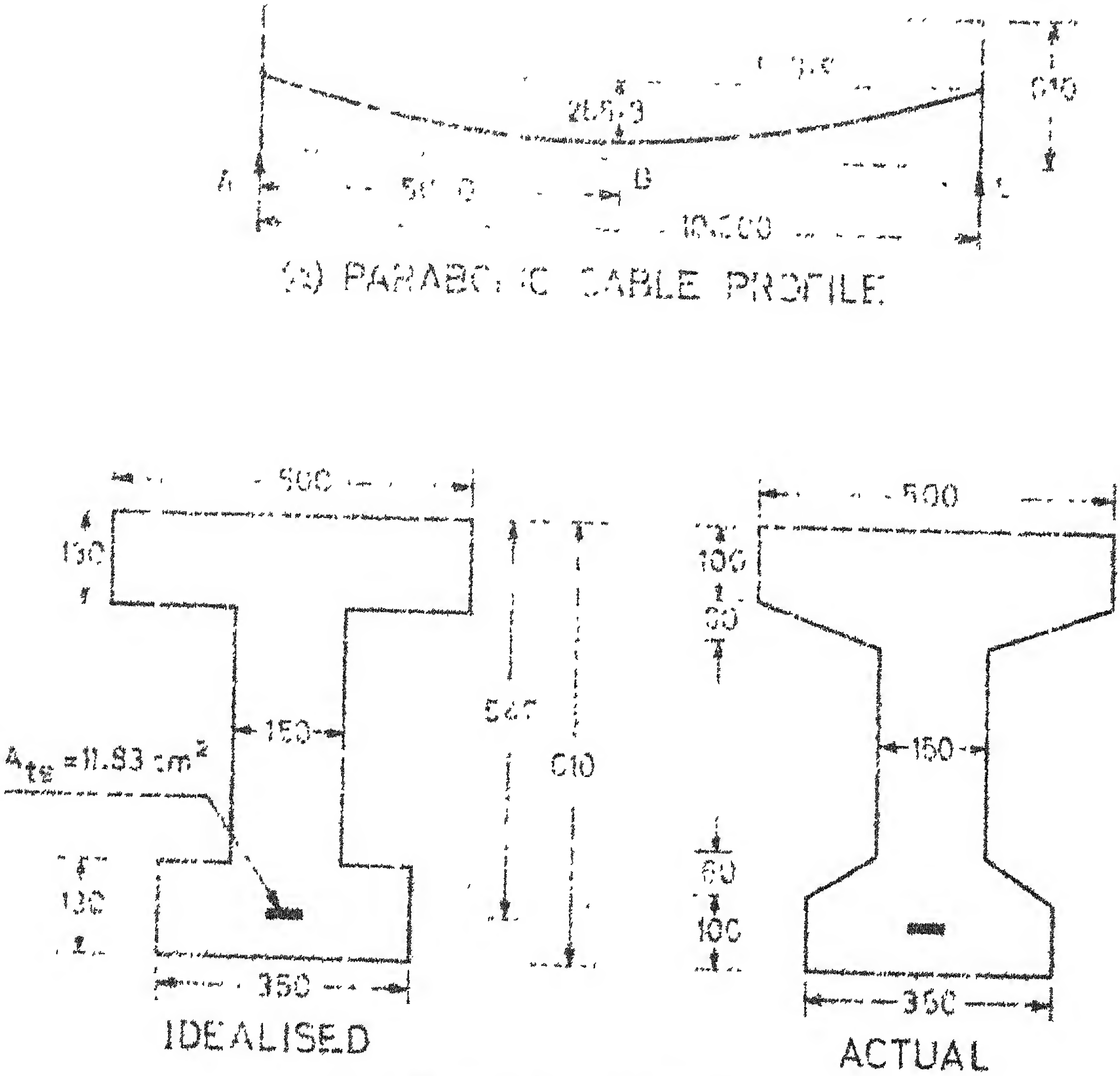
(ii) Using Monte Carlo method, generate samples for κ of the section determined in the above step.

(iii) Using Eq. 5.11, compute p_t of the section.

The procedure is illustrated with an example.

Example 5.1

The simply supported PSC beam of Example 3.1 is subjected to a total dead load (including self weight of the beam) of 1500 kg/m. The beam is designed with concrete strength of 350 kg/cm^2 and steel strength of 15000 kg/cm^2. The effective span and area of steel are



(a) PARABOLIC CABLE PROFILE

(b) MIDSPAN SECTION

FIG 3.11 SIMPLY SUPPORTED BEAM-EXAMPLE 3.1

ALL DIMENSIONS ARE IN mm

The method of evaluation of probability of failure at this limit state for the above two cases is given below.

Case 1

Under the action of dead load and prestress, critical sections (other than end sections) of the beam are determined as explained in article 3.9. Each span can have a failure mode under the action of dead load and prestress and each critical section has a failure probability. Hence if there are K critical sections in the ith failure mode, the probability of occurrence of the failure mode is given by the probability of the intersection of events Z_k defining the failure of each critical section. The probability of occurrence of the mode i at the limit state of strength at transfer of prestress, p_{ti}, is given by

$$p_{ti} = P(Z_1 \cap Z_2 \cap - - - \cap Z_K) \tag{5.12}$$

Assuming events Z_k are independent,

$$p_{ti} = \prod_{k=1}^{K} p_{tz_k} \tag{5.13}$$

where p_{tz_k} is the value of p_t of the section Z_k. This can be evaluated as explained article 4.6. If there are m failure modes, bounds on p_{ts} is given by

$$\max\ p_{ti} \leq p_{ts} \leq \sum_{i=1}^{m} p_{ti} \tag{5.14}$$

where p_{ts} is the probability of failure of the continuous beam at limit state of strength at transfer of prestress.

Case 2

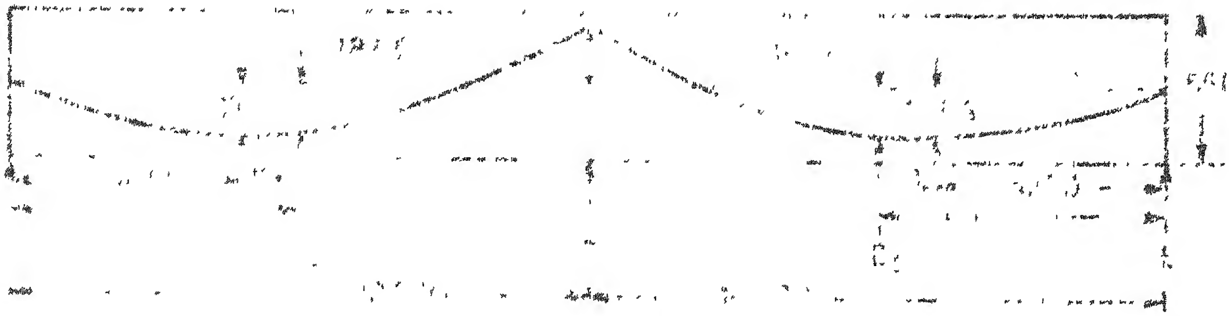
The complete collapse of the beam takes place when first yielding occurs at any one of the end sections. The value of p_t of this section can be computed as explained in article 5.5. The value of p_{ts} of the beam for this case is equal to p_t of the end section.

The procedure is illustrated with examples.

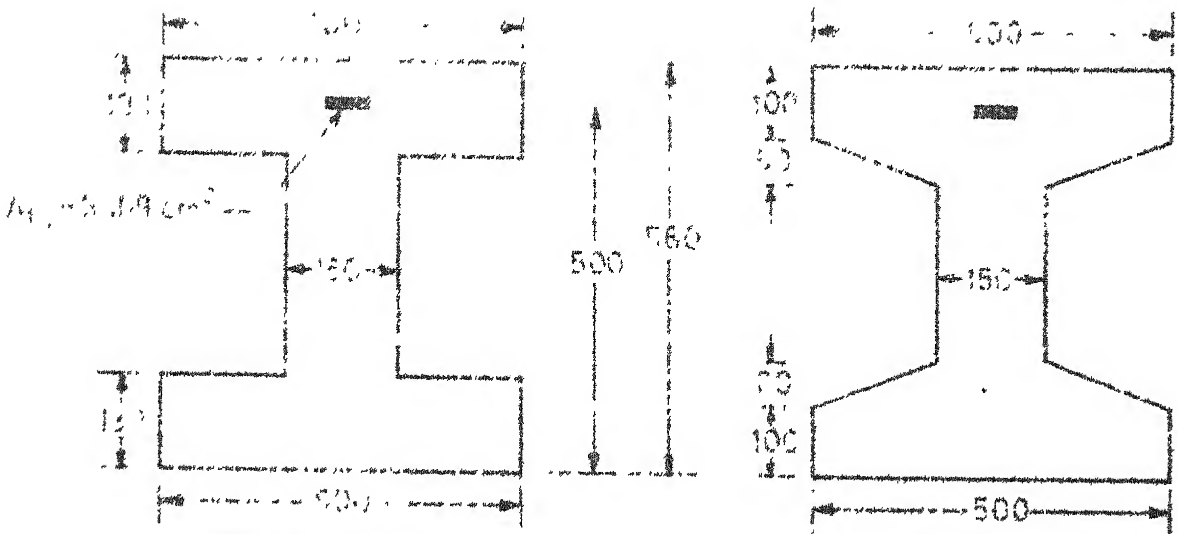
Example 5.2

The two span continuous PSC beam of Example 3.2 is subjected to a total dead load of 1500 kg/m (including self weight of the beam). The beam is designed with concrete strength of 350 kg/cm^2 and steel strength of 15000 kg/cm^2. The effective span and area of steel are 10 m and 10.89 cm^2. The cross section of the beam and cable profile are shown in Fig. 3.12 which is again given here for ready reference. The end sections (A and E) are rectangular with dimensions 50cm × 56cm. Parameters of σ_{cu} and σ_s remain same as in Example 3.2.

Critical sections of the beam for case (i) failure are C, B_1 and D_1 (Fig. 3.12) and section A(or E) for case (ii) failure. Dead load bending moments at sections C, B_1 and D_1 are 18.75tm, 10.39tm and 10.39tm respectively. Using distributions and parameters of σ_{cu} and σ_s and Eqs. 5.4 to 5.10, 10000 samples are generated for κ of sections C, B_1(or D_1) and A by Monte Carlo method. Parameters of κ of the above sections obtained from Monte Carlo method are given below.



(a) PARABOLIC CABLE PROFILE



IDEALISED

ACTUAL

(b) SECTION AT C

FIG.3.12 TWO SPAN CONTINUOUS BEAM-EXAMPLE 3.2

ALL DIMENSIONS ARE IN mm

Section C : $\kappa_m = 2.738$; $s_\kappa = 0.386$

Section B_1(or D_1) : $\kappa_m = 2.416$; $s_\kappa = 0.332$

Section A (or E) : $\kappa_m = 5.918$; $s_\kappa = 0.806$

Case 1 failure

Using Eq. 5.11, the value of p_t of the section C, p_{tC}, is

$$p_{tC} = \phi\left(\frac{1.0 - 2.738}{0.386}\right) = 3.4 \times 10^{-6}$$

and section B_1 is

$$p_{tB_1} = \phi\left(\frac{1.0 - 2.416}{0.332}\right) = 1.0 \times 10^{-5}$$

Using Eq. 5.13, the probability of occurrence of the failure mode in span AC, p_{tAC}, is

$$p_{tAC} = p_{tC} \cdot p_{tB_1} = 3.4 \times 10^{-11}$$

The value of p_{tCE} is equal to p_{tAC}.

Bounds on the value of p_{ts}, using Eq. 5.14, is given by

$$3.4 \times 10^{-11} \leq p_{ts} \leq 6.8 \times 10^{-11}$$

Case 2 failure

The value of p_t of the section A is computed using Eq. 5.11 and equal to 5×10^{-10}. This is higher than the value of p_{ts} obtained from case (i) failure. Hence the probability of the failure of the beam at limit state of strength at transfer of prestress is equal to 5×10^{-10}.

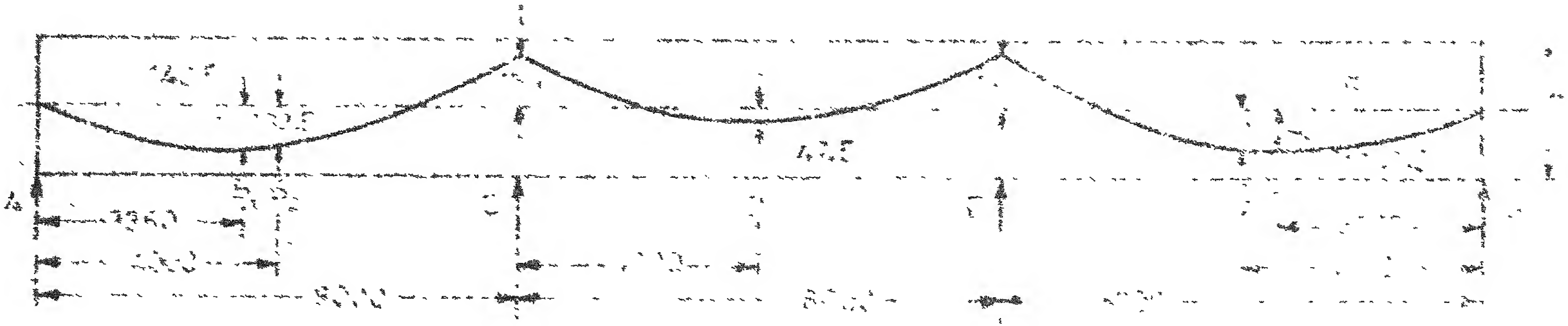
Example 5.3

The three span continuous PSC beam of Example 3.3 is subjected to a total dead load (including self weight of the beam) of 1000kg/m. σ_{cu} and σ_s are distributed as $N(422.8, 56)$kg/cm^2 and $N(15680,488)$kg/cm^2 respectively. The effective span and area of steel are 8m and 6.94cm^2 respectively. The cross section of the beam and cable profile are shown in Fig. 3.13 which is again given here for ready reference. The end sections A and G are rectangular with dimensions 40cm × 44cm.

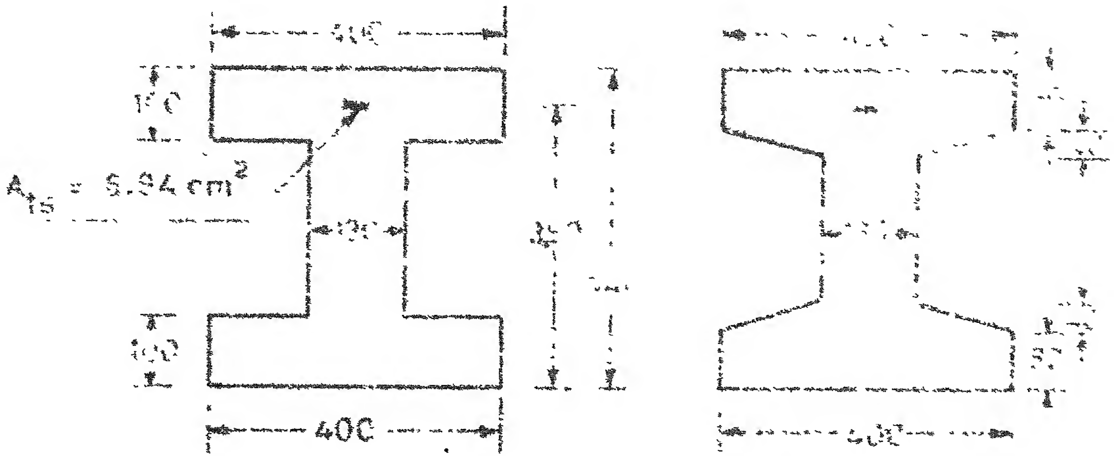
The procedure is same as illustrated in the previous example. Critical sections of the beam for case (i) failure are B_1, C, D, E and F_1 (Fig. 3.13) and section A or G for case (ii) failure. The values of dead load bending moments at the above critical sections are taken from table 3.20. Using Monte Carlo method, 10000 samples are generated for κ of sections B_1,C, D and A and parameters of κ obtained from the above method are given in table 5.1. The calculated values of p_t and the bounds on p_{ts} obtained as illustrated in the previous example are also given in the same table. From table 5.1, the bounds on p_{ts} for case (i) failure are,

$$1.04 \times 10^{-10} \leq p_{ts} \leq 2.08 \times 10^{-10}$$

For case (ii) failure, the value of p_t of the section A is computed and is equal to 4.1×10^{-10}. This is greater than the upper bound value of 2.08×10^{-10} for case(i) failure. Hence the probability of failure of the beam at limit state of strength at transfer of prestress is equal to 4.1×10^{-10}.



(a) PARABOLIC CABLE PROFILE



IDEALISED

ACTUAL

(b) SECTION AT C

FIG.3.13 THREE SPAN CONTINUOUS BEAM-EXAMPLE 3.3

ALL DIMENSIONS ARE IN mm

Table 5.1. Reliability analysis of the beam of Fig. 3.13 at limit state of strength at transfer for PV of σ_{ou} and σ_s

Critical Section	κ_m	s_κ	p_t	Probability of Occurrence of Failure Mode in Span: AC	CE	EG	Bounds on p_{ts}
Case (i)							
B_1 & F_1	2.523	0.351	6.70(-6)				
C & E	2.348	0.324	1.55(-5)				
D	3.217	0.442	2.70(-5)				
				1.04(-10)*	6.50(-15)	1.04(-10)	1.04(-10) $\leq p_{ts} \leq$ 2.08(-10)
Case (ii)							
A(or E)	5.286	0.786	4.1(-10)				

Probability of failure of the beam at limit state of strength at transfer of prestress = 4.1(-10)

* 1.04(-10) is read as 1.04×10^{-10}.

5.7 PROBABILITY DISTRIBUTION OF κ FOR PROBABILISTIC VARIATIONS OF σ_{cu} , σ_s AND DIMENSIONS OF SECTION

In the previous articles of this Chapter, only probabilistic variations of σ_{cu} and σ_s have been considered in the reliability analysis of PSC beams at limit state of strength at transfer of prestress. Random variations of dimensions b_t, b_b, t_t, t_b, d, b', h and D_s of section are also now taken into account in the determination of probability distribution and parameters of κ . The PSC section shown in Fig. 3.11 is considered. The mean and standard deviations of σ_{cu}, σ_s, b_t, b_b, t_t, t_b, b', h, d and D_s are available in Example 3.4. Using normal distributions and parameters of all the above random variables 10000 samples are generated for κ of the section by Monte Carlo method. The histogram of the generated data is given in Fig. 5.3. It is found that normal, lognormal, Type I extremal (largest), Type II extremal (largest) distributions do not satisfy chi square test at one percent level. However normal and lognormal distributions were suggested by Cornell, Ang and Lind (37, 38, 39) for (R/S) in the risk based evaluation analysis of R.C.C. beams.

In this chapter κ represents the ratio of (R/S). Hence, as already normal distribution has been observed for κ and used in article 5.4 for probabilistic variations of σ_{cu} and σ_s, here also to be in consistent with, the same distribution is adopted for κ for probabilistic variations of σ_{cu}, σ_s and dimensions of section.

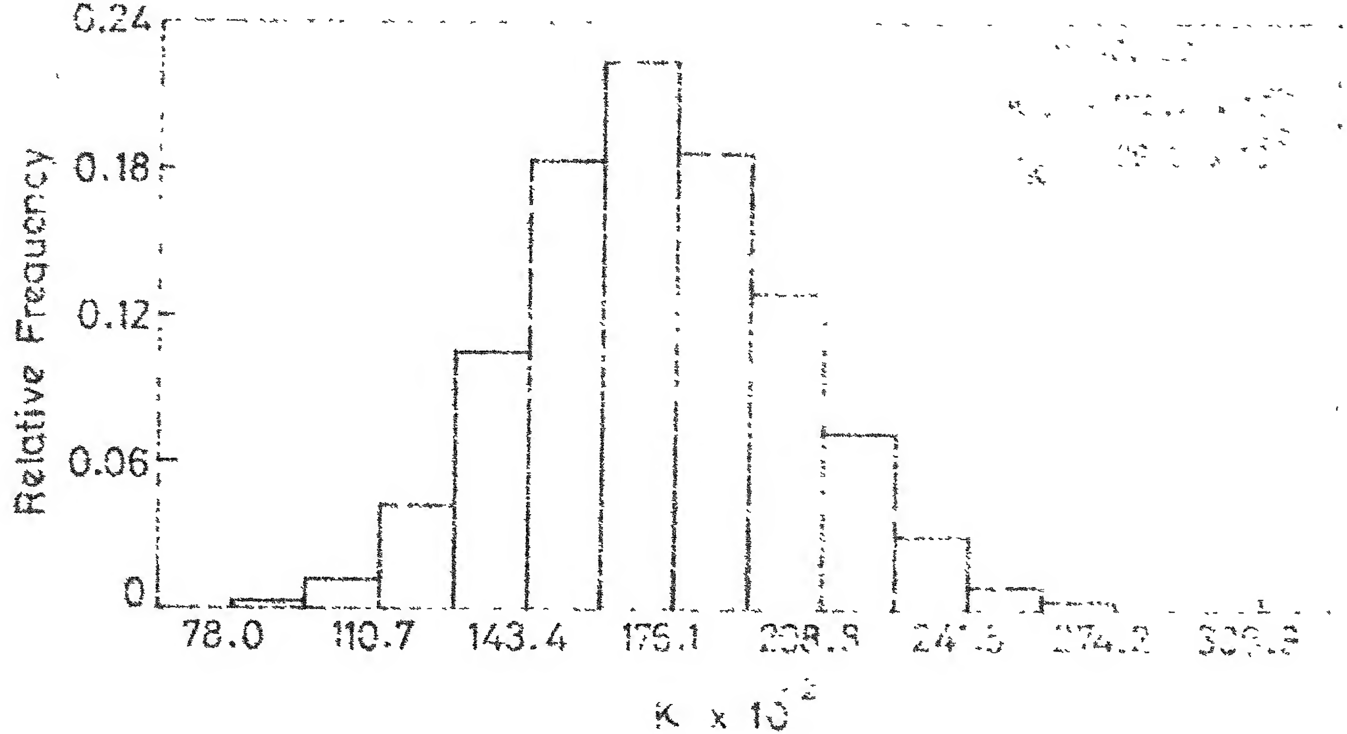


FIG.5.3 HISTOGRAM OF K FOR PV OF σ_{cu}, σ_{s} AND DIMENSIONS OF SECTION

For the assumed normal distribution, the value of p_t can be computed by Eq. 5.11. The procedure for the reliability analysis of a simply supported PSC beam is same as explained in article 5.5. This is illustrated with an example.

Example 5.4

The simply supported PSC beam of Example 3.4 (Fig. 3.11) is considered with probabilistic variations of σ_{cu}, σ_s and dimensions of section. Other particulars of the beam and parameters of all random variables are same as in Example 3.4.

The critical section is the midspan of the beam. For this section, samples have already been generated for κ and shown in Fig. 5.3. From the same figure the following values are taken.

$$\kappa_m = 1.723 \quad \text{and} \quad s_\kappa = 0.296$$

The computed value of p_t, using Eq. 5.11, is 7.5×10^{-3}.

The probability failure analysis of continuous PSC beams for probabilistic variations of σ_{cu}, σ_s and dimensions of section is same as explained in article 5.7 and illustrated in Examples 5.2 and 5.3 except that random variations of dimensions of section are also to be taken into account. The parameters of the random variables of the sections of two span (Fig. 3.12) and three span (Fig. 3.13) beams are available in tables 3.9 and 3.13. The results of the reliability analysis of the above beams at limit state of strength at transfer of prestress for probabilistic variations of σ_{cu}, σ_s and dimensions of section are presented in tables 5.2 and 5.3.

Table 5.2. Reliability analysis of the beam of Fig. 3.12 at limit state of strength at transfer for PV of σ_{cu}, σ_s and dimensions of section

Critical Section	κ_m	s_κ	p_t	Probability of Occurrence of Failure Mode in Span		Bounds on p_{ts}
				AC	CE	
Case (i)						
B_1 & D_1	2.229	0.372	8.2(-5)*			
C	2.578	0.419	2.6(-4)			
				2.13(-8)	2.13(-8)	$2.13(-8) \leq p_{ts} \leq 4.26(-8)$
Case (ii)						
A or E	5.867	0.838	2.95(-9)			

The probability of failure of the beam at limit state of strength at transfer of prestress : $2.13(-8) \leq p_{ts} \leq 4.26(-8)$

* 8.2(-5) is read as 8.2×10^{-5}.

Table 5.3. Reliability analysis of the beam of Fig. 3.13 at limit state of strength at transfer for PV of σ_{cu}, σ_s and dimensions of section

Critical Section	κ_m	s_κ	p_t	Probability of Occurrence of Failure Mode in Span			Bounds on p_{ts}
				AC	CE	EG	
Case (i)							
B_1 & F_1	2.413	0.383	1.12(-4)*				
C & E	2.192	0.372	6.60(-4)				
D	3.175	0.469	1.83(-6)				
				7.39(-8)	7.92(-13)	7.39(-8)	7.39(-8) $\leq p_{ts} \leq$ 14.78(-8)
Case (ii)							
A(or G)	5.776	0.825	3.3(-9)				

The probability of failure of the beam at limit state of strength at transfer of prestress : 7.39(-8) $\leq p_{ts} \leq$ 14.78(-8)

* 1.12(-4) is read as 1.12 $\times 10^{-4}$.

CHAPTER 6

RELIABILITY BASED DESIGN OF PRESTRESSED CONCRETE BEAMS FOR LIMIT STATE OF STRENGTH

6.1 INTRODUCTION

The design of simply supported and continuous PSC beams for a given probability of failure for limit state of strength is presented in this chapter. A simple reliability based design method is given for two cases considering probabilistic variations of:

(i) live load and strengths of materials and

(ii) live load, strengths of materials and geometric properties of section.

The reliability based design formulation reduces to the problem of solving an integral equation and finding out the required mean value of the resistance of section for given probability of failure and known coefficient of variation (CV) of the resisting moment.

6.2 RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED BEAMS

6.2.1 Introduction

The probability of failure of a simply supported beam is equal to the probability of failure of its critical section. The probability of fa

p_{fo} '

$$p_{fu} = P(F|U)\ P(U) \tag{6.1}$$

$$p_{fo} = P(F|O)\ P(O) \tag{6.2}$$

p_{fu} and p_{fo} are probabilities of failure of the section for under-reinforced and over-reinforced cases respectively. In any design problem, the p_f of the beam only is specified. If Q and M_{ru} are lognormally and normally distributed and p_{fu} and coefficient of variation of M_{ru} are known, the only unknown in Fig. 3.51 is M_{rum}. The integral equation can be solved and the value of M_{rum} be obtained. A section is to be proportioned for this mean value of M_{ru}.

6.2.2 Reliability Based Design Method for Probabilistic Variations of Live Load and Strengths of Materials

In Chapter 3, the values of p_{fu}, p_{fo} and p_f of each critical section of the different beams (Figs. 3.11 to 3.13) for different loading cases have been computed. These values are listed in table 6.1. The values of λ_s, which is defined as the ratio of p_{fu} to p_f, for each case is calculated and given in the same table. The average value of this ratio is found to be 0.3477. The coefficient of variation of M_{ru} of each critical section of the beams in Examples 3.1, 3.2 and 3.3 are also given in the same table 6.1. It can be seen that coefficient of variation of M_{ru} is almost same in all cases and the average value of coefficient of variation of M_{ru} is 0.03097 which is approximately equal to the coefficient of variation of strength of steel, λ_s, whose value is 0.0311.

Table 6.1. Values of λ_s and CV of M_{ru} for different sections for PV of Q, σ_{cu} and σ_s

Sl. No.	Beam in	Section	p_{fu}	p_{fo}	p_f	λ_s	CV of M_{ru}
1	Example 3.1	Midspan	5.74(-10)*	1.11(-9)	1.68(-9)	0.3413	0.0314
2	Example 3.2	B_1	2.41(-13)	3.77(-13)	6.18(-13)	0.39	0.0310
3		B_2	1.29(-10)	1.42(-10)	2.71(-10)	0.476	0.0307
4		C	2.57(-8)	2.18(-8)	5.23(-8)	0.491	0.0311
5		C	1.30(-13)	2.53(-13)	3.83(-13)	0.339	
6	Example 3.3	B_1	1.1(-11)	2.1(-11)	3.2(-11)	0.3363	0.0309
7		B_1	1.9(-12)	4.9(-12)	5.9(-12)	0.314	
8		B_2	1.1(-9)	1.9(-9)	3.0(-9)	0.373	0.0311
9		B_2	2.1(-10)	3.9(-10)	6.0(-10)	0.349	
10		C	2.1(-16)	6.2(-16)	8.3(-16)	0.252	0.0309
11		C	1.2(-13)	2.2(-13)	3.4(-13)	0.343	
12		C	2.8(-9)	3.0(-9)	5.8(-9)	0.49	
13		C	2.1(-10)	2.5(-10)	4.6(-10)	0.449	
14		D	1.4(-13)	11.0(-13)	12.4(-13)	0.112	0.0309
15		D	0.24(-9)	1.3(-9)	1.54(-9)	0.161	

Average value of $\lambda_s = 0.3477$

Average value of CV of $M_{ru} = 0.03097$

* 5.74(-10) is read as 5.74×10^{-10}.

Having established the values of λ_s and coefficient of variation of M_{ru}, the following are the steps for the reliability based design of simply supported beams for limit state of strength.

(i) For given p_f of the beam, calculate p_{fu} by multiplying p_f by λ_s.

(ii) Using the value of coefficient of variation of M_{ru} as 0.031 obtain s_{ru} in terms of M_{rum}.

(iii) Substituting above values of p_{fu} and s_{ru} in Eq. 3.51, solve the integral equation and obtain M_{rum}.

(iv) If the section is given, find out the value of area of steel for the value of M_{rum} obtained in the previous step.

It is assumed that the section is given and the area of steel is to be found out. For flanged sections, the value of M_{ru} is given by two Eqs. 3.13 and 3.15 depending on whether neutral axis is in the flange or web.

If neutral axis lies in the flange,

$$M_{rum} = \sigma_{sm} A_{ts} d \left(1 - \frac{0.75 A_{ts} \sigma_{sm}}{b_t d \sigma_{cum}}\right) \tag{6.3}$$

For given section and M_{rum}, the above equation is a quadratic equation of A_{ts}. The quadratic equation can be solved and the value of A_{ts} is given by

$$A_{ts} = \frac{1}{2D_1} \left(1 - \sqrt{1 - 4D_1 D_2} \right) \tag{6.4}$$

where,

$$D_1 = \frac{0.75\ \sigma_{sm}}{b_t d\ \sigma_{cum}}$$

$$D_2 = \frac{M_{rum}}{d\ \sigma_{sm}}\ .$$

If the neutral axis lies in the web,

$$M_{rum} = 0.7\ \sigma_{cum}(b_t - b')t_t(d - 0.5t_t) + \sigma_{sm}\ A_{tsw}\ d\left(1 - \frac{0.75\ A_{tsw}\ \sigma_{sm}}{b'd\ \sigma_{cum}}\right) \qquad (6.5)$$

This is also a quadratic equation in A_{tsw} if M_{rum} and sectional properties are known. The value of A_{tsw} is given by

$$A_{tsw} = \frac{1}{2D_3}\left(1 - \sqrt{1 - 4D_3D_4}\right) \qquad (6.6)$$

where,

$$D_3 = \frac{0.75\ \sigma_{sm}}{b'd\,\sigma_{cum}}$$

$$D_4 = [M_{rum} - 0.7\ \sigma_{cum}(b_t - b')t_t(d-0.5t_t)]\ /(d\sigma_{sm})$$

Total area of steel is given by

$$A_{ts} = A_{tsw} + A_{tsf} \qquad (6.7)$$

where,

$$A_{tsf} = \frac{0.68\ \sigma_{cum}(b_t - b')t_t}{\sigma_{sm}}\ . \qquad (6.8)$$

Initially it may be assumed that neutral axis lies in the web and the value of A_{tsw} may be calculated. If this is negative, A_{ts} is given by Eq. 6.2

The method is illustrated by an example.

Example 6.1

A simply supported prestressed concrete beam of effective span 10 m is subjected to a dead load of 1500 kg/m (including self weight of beam) and probabilistic live load distributed as LN(608.85 kg/m, 0.368). σ_{cu} and σ_s are distributed as N(422.8,56) kg/cm^2 and N(15680,488) kg/cm^2 respectively. The section of the beam is given in Fig. 3.11. Determine the area of steel if the p_f of the beam is 10^{-8}.

The value of p_{fu} is obtained as

$$p_{fu} = \lambda_s \, p_f = 0.3477 \times 10^{-8}$$

using coefficient of variation of $M_{ru} = 0.031$

$$s_{ru} = 0.031 \, M_{rum}$$

Substituting p_{fu} and s_{ru} in Eq. 3.51, the equation becomes

$$0.3477 \times 10^{-8} = \int_{-\infty}^{\infty} e^{-w} \frac{1}{\sqrt{\pi}} \left[1 - \phi \left\{ \frac{\log\left(\dfrac{\sqrt{2}w \; 0.031 M_{rum} + M_{rum} - M_g}{M_{qm}} \right)}{s_q} \right\}\right] dw \tag{6.9}$$

The values of M_{qm}, M_g and s_q are 7.61 tm, 18.75 tm and 0.368. The above equation is solved in the following way. The iteration is started with an initial value of M_{rum} and the right hand side portion of the Eq. 6.9 is evaluated using numerical quadrature method (illustrated in Chapter 3). Depending on whether the evaluated value

is smaller or greater than the value on the left hand side of the equation, the value of M_{rum} is decreased or increased correspondingly in the next iteration. Iteration is stopped when the difference between the values on the right hand side and left hand side of the Eq. 6.9 is less than 0.008 percent of the left hand side value of the equation. Using the above procedure, the Eq. 6.9 is solved and the value of M_{rum} is found to be 83.82 tm.

Using Eq. 6.6, the value of A_{tsw} is obtained as $2.4 cm^2$. Since this is positive, the neutral axis lies in the web. The value of A_{tsf} is computed using Eq. 6.8 and is equal to $8.34\ cm^2$. Hence the value of

$$A_{ts} = A_{tsw} + A_{tsf} = 10.7\ cm^2$$

Comparing this with Example 3.5, it can be seen that for an area of steel of $11.83\ cm^2$, the p_f was 1.7×10^{-9}. Since in this problem p_f has been increased to 10^{-8}, the area of steel has been decreased as expected.

6.2.3 Reliability Based Design Method for Probabilistic Variations of Live Load, Strengths of Materials and Dimensions of Section

Design procedure is same as explained in the previous article except that values of λ_{sd} and coefficient of variation of M_{ru} are different. λ_{sd} denotes the ratio of p_{fu} to p_f for probabilistic variations of live load, strengths of materials and dimensions of section. The values of p_{fu} and p_f, calculated for different critical sections of beams for different loading cases, are listed

in table 6.2. The average value of λ_{sd} is found to be 0.5847. Similarly the values of coefficient of variation of M_{ru}, obtained by Monte Carlo method, for the same sections for probabilistic variations of σ_{cu}, σ_s, b_t, b', t_t, d and D_s in Chapter 3 are also given in the same table. It is found that the values of coefficient of variation of M_{ru} are almost same in all cases and the average value is 0.0543. This is valid for the values of the parameters of the above random variables used in this thesis. In a general case, for given mean values and standard deviations of the random variables σ_{cu}, σ_s, b_t, b', t_t, d and D_s, the mean value and standard deviation of M_{ru} can be found by applying partial derivative method (59) to Eq. 3.13 or 3.15 as both yield almost same value.

Using the above established values of λ_{sd} and coefficient of variation of M_{ru}, the value of M_{rum} is computed for given p_f of the beam as explained in the previous article. In this case also the same equations (i.e. Eqs. 6.3 to 6.8) are valid except that in every equation mean values of the random variables b_t, b', t_t, and d are to be used along with σ_{cum} and σ_{sm}. The procedure is illustrated with an example.

Example 6.2

The simply supported PSC beam of Example 6.1 is considered. It is subjected to probabilistic variations of live load, strengths of materials and dimensions of section. Distributions and parameters

Table 6.2. Values of λ_{sd} and CV of M_{ru} for different sections for PV of Q, σ_{cu}, σ_s and dimensions of section

Sl. No.	Beam in	Section	p_{fu}	p_{fo}	p_f	λ_{sd}	CV of M_{ru}
1.	Example 3.1	Midspan	1.08(-10)*	0.76(-10)	1.84(-10)	0.586	0.0534
2	Example 3.2	B_1	3.95(-14)	2.20(-14)	6.15(-14)	0.642	0.0537
3		B_2	2.67(-11)	1.22(-11)	3.89(-11)	0.686	0.0547
4		C	4.97(-9)	1.56(-9)	6.53(-9)	0.761	0.0545
5		C	1.82(-14)	1.09(-14)	2.91(-14)	0.625	
6	Example 3.3	B_1	2.1(-12)	1.4(-12)	3.49(-12)	0.587	0.0548
7		B_1	3.4(-13)	2.6(-13)	5.62(-13)	0.605	
8		B_2	2.4(-10)	1.5(-10)	3.88(-10)	0.616	0.0539
9		B_2	4.2(-11)	2.9(-11)	7.07(-11)	0.593	
10		C	1.8(-17)	2.6(-17)	4.43(-17)	0.413	0.0545
11		C	1.9(-14)	1.2(-14)	3.11(-14)	0.621	
12		C	6.1(-10)	2.3(-10)	8.35(-10)	0.726	
13		C	4.1(-11)	1.8(-11)	5.88(-11)	0.697	
14		D	2.6(-14)	8.5(-14)	11.12(-14)	0.232	0.0550
15		D	8.1(-11)	13.2(-11)	21.3(-11)	0.380	

Average value of λ_{sd} = 0.5847

Average value of CV of M_{ru} = 0.0543

* 1.08(-10) is read as 1.08×10^{-10}.

of Q, σ_{cu} and σ_s remain same as in Example 6.1. The parameters and distributions of random variables b_t, b', t_t and d of this beam are available in Example 3.4. Determine the mean value of area of steel if the p_f of the beam is 10^{-8}.

Using λ_{sd} = 0.5847 (established earlier), the value of $p_{fu} = 0.5847 \times 10^{-8}$.

Using coefficient of variation of M_{ru} = 0.0543 (established earlier),

$$s_{ru} = 0.0543\ M_{rum}$$

Substituting p_{fu} and s_{ru} in Eq. 3.51 and solving the same, the value of M_{rum} is obtained as 83.66 tm. Substituting the mean values of b_t, b', t_t and d in Eq. 6.5, the mean value of A_{tsw} is computed as 1.12 cm^2. Since this is positive, mean value of neutral axis lies in web. Using Eq. 6.8, the mean value of A_{tsf} is obtained as 8.63 cm^2. Hence A_{tsm}, the mean value of area of steel, is given by

$$A_{tsm} = 1.12 + 8.63 = 9.75\ cm^2$$

This is less than the value of steel used in the Example 3.6. The beam has been designed in this problem for higher value of p_f and as expected, less area of steel is required.

6.3 RELIABILITY BASED DESIGN OF CONTINUOUS BEAMS

6.3.1 Introduction

Continuous beams have a number of failure modes and are subjected to different live load conditions. Taking a two span

continuous beam, there are three live load conditions which give maximum positive and negative bending moments in the beam. Even though other load conditions i.e. partial loading of a span or spans are possible, the failure of the beam for such conditions will be very very small and insignificant compared to the values of probability of failure of the beam for the three load conditions. Hence, considering only 3 loading cases as in Example 3.2, it is seen that the probability of failure of the beam for all load conditions and failure modes vary from one to two times the value of lower bound (LB) of p_{fs}. It is also noticed that LB value of p_{fs} corresponds to the probability of occurrence of a particular failure mode for the most critical loading condition. Hence, if it is possible to find out the probability of occurrence of the particular mode, for which probability of failure is maximum under the most critical loading condition, from given p_{fs} of the beam, the beam can be designed for this failure mode and its probability of failure.

6.3.2 Reliability Based Design Method for Continuous Prestressed Concrete Beams

These are the following steps that are associated in the reliability based design of continuous prestressed concrete beams for limit state of strength.

(i) Assume the lower bound value of probability of failure of the beam equal to the given p_{fs} of the beam (This is on the conservative side).

(ii) Find out the most critical loading condition for the given beam which gives maximum moment at any critical section.
Find out the critical failure mode, i.e. the probability of occurrence of which is the highest, and its critical sections for this loading case. The probability of occurrence of this failure mode is assumed as the value of p_{fs}.

(iii) Fix the probability of failure of the section which yields first (this is fixed as 10^{-4} or 10^{-5}) in the critical failure mode. Hence the probability of failure of the other critical section in the same failure mode (say, there are two critical sections in the failure mode) is the probability of occurrence of the failure mode divided by the probability of failure of the section yielding first

(iv) Assume effective depth of the beam at the first yielding critical section (this is fixed as maximum as possible).
Design the section, i.e. A_{ts}, for probability of failure of the section, fixed in step (iii), based on the method explained in article 6.2.2.

(v) For the calculated probability of failure of the second critical section, find out the value of M_{rum} of the section as explained in article 6.2.2. For this value of M_{rum} and calculated A_{ts} in step (iv), find out the effective depth of the section using Eqs. 6.3 or 6.5 whichever is applicable. Adopt a parabolic cable profile passing through the cable positions fixed at the critical sections.

The procedure is illustrated with examples.

Example 6.3

A continuous prestressed concrete beam having two equal spans is subjected to a dead load of 1500 kg/m (including self weight of the beam) and probabilistic live load distributed as LN(608.85 kg/m, 0.368). σ_{cu} and σ_s are distributed as N(422.8,56) kg/cm^2 and N(15680,498) kg/cm^2 respectively. Other particulars remain same as in Example 3.2 (Fig. 3.12). The probability of failure of the beam is fixed as 10^{-11}.

The LB value of the p_{fs} is taken as equal to the given probability of failure of the beam i.e. 10^{-11}.

The critical loading condition for the beam is when both spans are fully occupied by live load. The first failure is at section C. The critical failure mode occurs in span AC or CE. Considering failure mode in span AC, the two critical sections are C and B_1 (Refer Example 3.2). The probability of occurrence of the failure mode in span AC, p_{fAC}, is taken as equal to the LB value of p_{fs}. Hence

$$p_{fAC} = 10^{-11}$$

Fixing p_{fC} as 10^{-4}, the probability of failure of the section B_1 is

$$p_{fB_1} = 10^{-11}/10^{-4} = 10^{-7}$$

Fixing effective depth of beam at C as 50 cms, the area of steel required for p_{fC} equal to 10^{-4} is 7.23 cm^2(obtained as illustrated in Example 6.1).

Using $p_{fB_1} = 10^{-7}$ (calculated earlier), the value of M_{rum} from Eq. 3.51 is computed as illustrated in Example 6.1 and is equal to 41.48tm. Using Eq. 6.3,

$$41.84 \times 10^5 = A_{ts}\ \sigma_{sm}\ d\ [1 - \frac{0.75\ A_{ts}\ \sigma_{sm}}{b_t d\ \sigma_{cum}}]$$

Substituting the values of A_{ts}, σ_{sm}, b_t and σ_{cum}, the value of d is obtained as 44.92cm. Using this d at section B_1, the section is checked whether neutral axis lies in the flange. It has been checked and found that neutral axis lies in the flange. A parabolic cable profile is adopted.

Example 6.4

A continuous prestressed concrete beam having three equal spans is subjected to a dead load of 1000 kg/m (including self weight of the beam) and probabilistic live load distributed as LN(487.08 kg/m, 0.368). Mean and standard deviations of σ_{cu} and σ_s and other particulars of the beam remain same as in Example 3.3 (Fig. 3.13). The probability of failure of the beam for all load conditions and failure mode is fixed as 10^{-11}.

The lower bound value of the p_{fs} of the beam is taken as equal to 10^{-11}.

The critical loading condition is when live load occupies two adjacent spans say AC and CE. For this loading condition the critical failure mode occurs in span AC. The critical sections in this failure mode are B_1 and C(Fig. 3.13). The probability of

occurrence of this failure mode in span AC is taken as equal to LB value of p_{fs} i.e. $p_{fAC} = 10^{-11}$.

Fixing $p_{fC} = 10^{-4}$ (Section C yields first), the probability of failure of the section B_1 is

$$p_{fB_1} = 10^{-7}$$

Fixing effective depth of beam at section C as 39 cm, the area of steel required for the value of p_{fC} equal to 10^{-4} is 3.93 cm^2.

Using the above value of A_{ts} and p_{fB_1}, the value of M_{rum} is computed, using Eq. 3.51 and method explained in article 6.2.2, as 23.42tm. The value of effective depth at B_1 for this M_{rum}, using Eq. 6.3, is obtained as 40.75cm. This is greater than the maximum possible effective depth i.e. 39 cm. This can be solved in two ways:

(i) By increasing the probability of failure of section C and thereby reducing the value of p_{fB_1}. This will require more area of steel for the same d_C and less d_{B_1}.

(ii) By using the maximum value of d_{B_1}, required A_{ts} for the value of M_{rum} = 23.42tm can be computed. This will result more area of steel and hence the probability of failure of section C will be less than 10^{-4} which will satisfy the design criteria.

By using the second method, the value of A_{ts} has been obtained as 4.11cm^2 for d_{B_1} = 39cm and M_{rum} = 23.42tm. A parabolic cable profile is adopted.

Similarly the same beams of Examples 6.3 and 6.4, subjected to probabilistic variations of live load, strengths of materials and dimensions of section, can be designed for a given probability of failure except that individual sections are to be designed based on the procedure explained in article 6.2.3.

The design method is simple. The solution is possible if the coefficient of variation of the resisting moment of the section for under-reinforced case is known. In the design formulation the mean value of the ultimate resisting moment of the section for under-reinforced case is first determined for given reliability of the beam. Assuming section is given, the method of finding area of steel for calculated M_{rum} has been illustrated. If a section is to be designed, it can be done using the ultimate strength design of a PSC section for required mean value of the resisting moment using mean values of σ_{cu} and σ_s.

CHAPTER 7

SEMI-PROBABILISTIC LIMIT STATE DESIGN OF PRESTRESSED CONCRETE BEAMS

7.1 Introduction

The philosophy in structural design is to provide a structure which can serve the required purpose under certain external forces with an assured degree of reliability.

Structural design methods can broadly be classified into the following:

1. Deterministic design method - In this method, design parameters, load and resistances are considered as deterministic and the safety is ensured by limiting action less than resistance.
2. Probabilistic design method (60) - In this method basic strength and load parameters are treated as random and the safety conditions are expressed by specifying a priori the probability that the structure will preserve its functional capacity over a given period of time.
3. Semi-probabilistic design method - In this method loads and resistances are fixed from statistical analysis and assigned probability of failure. The safety is ensured by limiting the action less than resistance.

In the semi-probabilistic limit state design method, safety is assured by the selection of various coefficients (for the strengths

of materials, loads etc.) dependent on the limit state considered. The values of these coefficients, wherever is possible, are proposed in this chapter based on the statistical analysis of materials and reliability analysis of PSC beams from Chapters 2 to 6. An example is presented illustrating the semi-probabilistic design method.

7.2 CHARACTERISTIC STRENGTHS OF MATERIALS

The characteristic strengths of materials are, by definition, those which have an agreed probability of not being obtained. It is given by

$$R_k = R_m - k\ s_r \tag{7.1}$$

where

R_k = characteristic strength

R_m = mean strength

s_r = standard deviation of strength

The CEB-FIP Committee recommends the value of k as 1.64 (5% probability) for strengths of concrete and steel. In Chapter 2 the value of k for σ_{ou} has been established as 1.6 for the field data. This is close to the value suggested by CEB-FIP Committee. Hence the same value, 1.64 (safer side), is recommended for k for strength of concrete. Hence

$$\sigma_{cuk} = \sigma_{cum} - 1.64\ s_c \tag{7.2}$$

Regarding the value of k for σ_s, the statistical analysis of field data on σ_s in Chapter 2, shows wide range of values for k varying from 0.08 to 12. Number of projects being less, more data

are necessary on σ_s to arrive at the value of k for σ_s for Indian conditions. Till then, the same value of 1.64 recommended by CEB-FIP Committee may be used. Hence

$$\sigma_{sk} = \sigma_{sm} - 1.64 \quad s_s \qquad (7.3)$$

7.3 DESIGN STRENGTHS OF MATERIALS

Reduction factors $\frac{1}{\gamma}$ are applied for various materials, to the characteristic strengths, to take into account the uncertain factors and those factors which cannot be easily evaluated and measured. For a given material the value of γ depends on the behaviour of material and seriousness of risk involved in attaining limit state. The calculated reduced strength, R_d, is called as the design strength:

$$R_d = \frac{R_k}{\gamma} \qquad (7.4)$$

CEB-FIP Committee recommends $\gamma = 1$ for σ_{cu} and σ_s at serviceability limit states and $\gamma = 1.15$ for σ_s and $\gamma = 1.4$ to 1.6 (depending on quality control) for σ_{cu} at ultimate limit states. The same values are used in this thesis.

7.4 CHARACTERISTIC LOADS

For loads where a possible increase could be detrimental the characteristic load, W_k, is defined by

$$W_k = W_m (1 + k\ \delta_w) \qquad (7.5)$$

where

W_m = the value of the most unfavourable loading with a fifty percent probability of its not being exceeded during the expected life of the structure.

k = coefficient depending on the agreed probability of loading greater than W_k.

δ_w = coefficient of variation of the maximum loading.

If on the other hand a reduction in the value of load will endanger the stability of the structure, the characteristic load, W_k', is defined as

$$W_k' = W_m' \, (1 - k\,\delta_w) \qquad (7.6)$$

where

W_m' = the value of the most unfavourable loading with a fifty percent probability of its not falling below in the expected life of the structure.

W_m is obtained from statistical analysis of loads on structures.

The load survey (Chapter 2) shows a mean live load of 130.33 kg/m^2 in office rooms at IIT, Kanpur. The I.S. Code (53) specifies 250 to 400 kg/m^2 for office buildings. Depending on this the value of k also varies. Local government and state and central government office structures in India are subjected to higher loads as compared to IIT, Kanpur buildings. Hence it will not be possible to fix the value of k unless extensive load data on similar office buildings are available. When adequate statistical data on loads are not available, the minimum loads specified by codes are taken as characteristic values.

7.5 DESIGN LOADS

Design loads, W_d, are obtained by multiplying the characteristic lo by load factors corresponding to particular limit state.

$$W_d = F\ W_k \tag{7.7}$$

where F is load factor.

P. Srinivasa Rao and C.S. Krishnamoorthy (61) discussed the criteria for fixing up the load factors for dead and live loads. Two partial load factors for live load, one for the variation of live load and the other for covering uncertainties over the exact distribution of live load, errors in analysis and for the accepted probability of failure, were introduced. The load factor for live load was expressed as the product of the above two partial load factors for live load. They concluded that even though the strength factors adopted are different in various codes, the load factors are recommended such that moment carrying capacities of PSC singly reinforced beams under working loads are found to be nearly the same.

Load factors differ from code to code to a limited extent. The actual margin of safety depends not only on load factors and material reduction factors but also on the quality control specifications. Even, if the load factors are large and if also the coefficient of variation of material strength is large, then the margin of safety is not proportional to load factors. A rational approach to load factors is to fix a probability of failure of the section and then arrive at the load factors. Probability of failure of a section can be stated as

$$P(M_r < M_{ew}) = p_f \tag{7.8}$$

As per ultimate strength design, the value of M_r is

$$M_r = M_u = F_g M_g + F_q M_q$$

or $$M_r = F_c(M_g + M_q) = F_c M_{ew} \qquad (7.9)$$

where F_c is the combined load factor.

Assuming normal distribution for resistance, the probability of failure can be expressed as

$$\phi\left(\frac{M_{ew} - M_{rm}}{s_r}\right) = p_f$$

$$\frac{M_{ew} - M_{rm}}{s_r} = \phi^{-1}(p_f) = -k$$

or $$M_{rm} = M_{ew} + k\ s_r \qquad (7.10)$$

Using $s_r = \delta_r M_{rm}$, the above equation is rewritten as

$$M_{rm}(1 - k\,\delta_r) = M_{ew} \qquad (7.11)$$

Substitution of Eq. 7.9 in 7.11 gives

$$F_c = \left(\frac{1}{1-k\ \delta_r}\right) \qquad (7.12)$$

Fig. 7.1 shows the variation of F_c and p_f for different values of δ_r.

Separate load factors for dead load and live load are obtained as follows. Using Eq. 7.11,

$$(F_g M_g + F_q M_q)(1 - k\ \delta_r) = M_g + M_q$$

$$F_q(1 - k\ \delta_r) = \frac{M_g}{M_q}(1 - F_g + k\delta_r F_g) + 1$$

or $$F_q = [1 + \nu(1 - F_g + k\delta_r F_g)] / (1 - k\delta_r) \qquad (7.13)$$

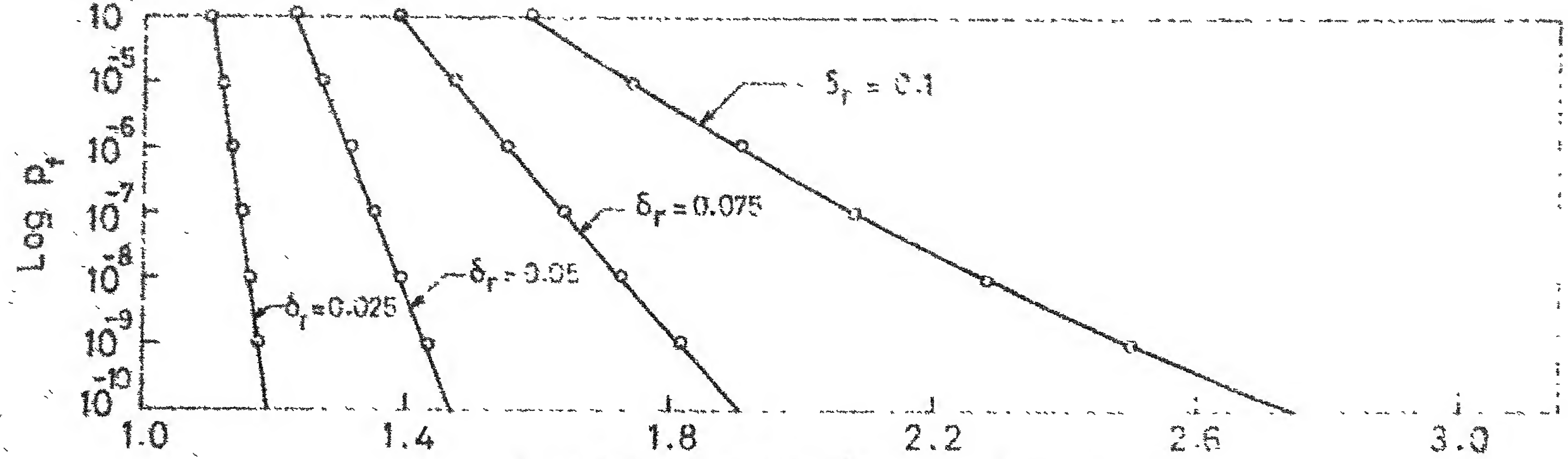


FIG. 7.1 VARIATION OF p_f AND F

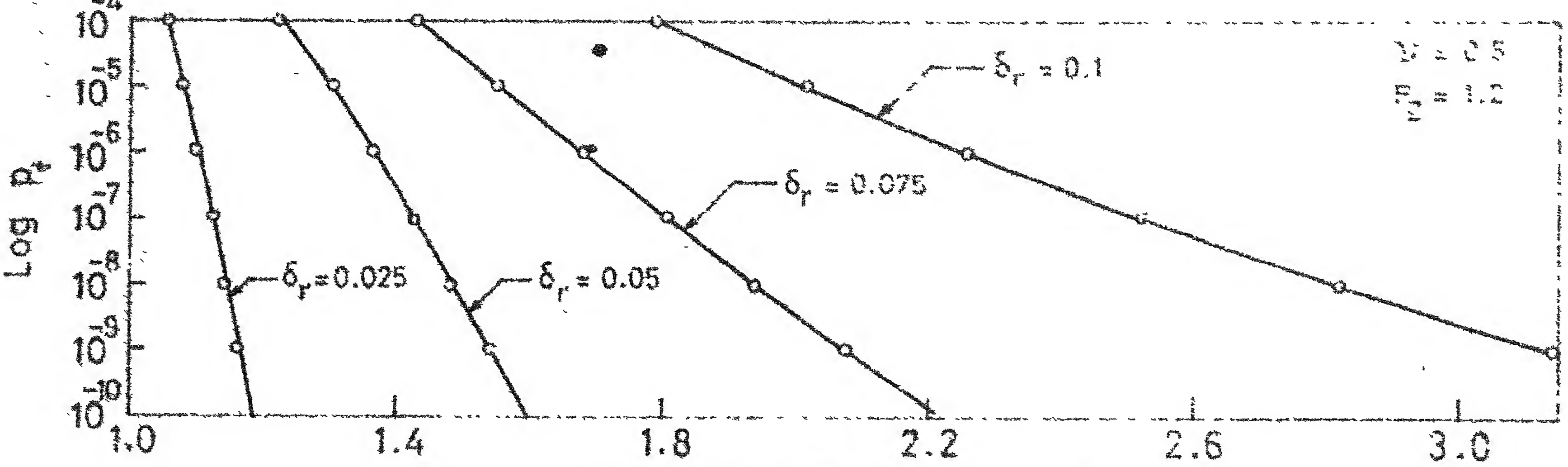


FIG. 7.2 VARIATION OF p_t AND F_q FOR $\nu = 0.5$ AND $F_s = 1.2$

where $\nu = (M_g/M_q)$. Hence load factor for live load depends on (i) the ratio of dead load bending moment to live load bending moment, (ii) load factor for dead load, (iii) the value of k which corresponds to the level of probability of failure fixed and (iv) coefficient of variation of resisting moment.

Figs. 7.2 to 7.4 show the variation of F_q and p_f. It can be seen from the above figures that

(i) for constant values of ν and F_g, F_q increases if (a) p_f decreases and(b) δ_r increases for constant value of p_f

(ii) for constant values of δ_r and F_g, F_q increases if (a) p_f decreases and (b) ν increases for constant p_f

(iii) for constant values of δ_r and ν, F_q increases if (a) p_f decreases and(b) F_g decreases for constant p_f.

From the reliability analysis of PSC beams (Chapter 3) at limit state of strength it is observed that coefficient of variation of ultimate resisting moment of a PSC section varies from 3 to 8 percent. If the random variations of geometric properties of a section are not considered, δ_r is about 3.1 percent for under-reinforced case and about 5.5 percent for over-reinforced case. Generally beams are designed as under-reinforced. However there is a probability of the same beams becoming over-reinforced. Hence δ_r is assumed as 5 percent for fixing load factor. In building floors, the value of ν varies from 0.3 to 1.5. Taking $\nu = 0.75$ and fixing $p_f = 10^{-6}$ and $F_g = 1.2$, the value of F_q, using Eq. 7.13, is found to be 1.4.

where $\nu = (M_g/M_q)$. Hence load factor for live load depends on (i) the ratio of dead load bending moment to live load bending moment, (ii) load factor for dead load, (iii) the value of k which corresponds to the level of probability of failure fixed and (iv) coefficient of variation of resisting moment.

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(ii) for constant values of δ_r and F_g, F_q increases if (a) p_f decreases and (b) ν increases for constant p_f

(iii) for constant values of δ_r and ν, F_q increases if (a) p_f decreases and(b) F_g decreases for constant p_f.

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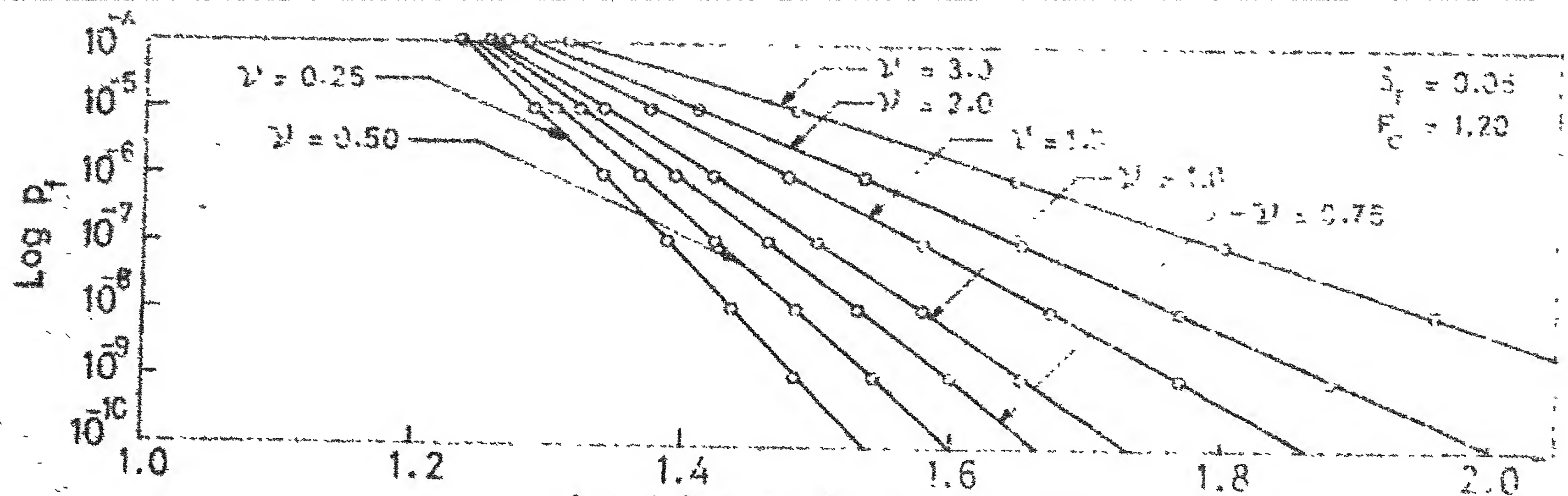


FIG. 7.3 VARIATION OF p_f AND F_g FOR δ_r = 0.05 AND F_c = 1.2

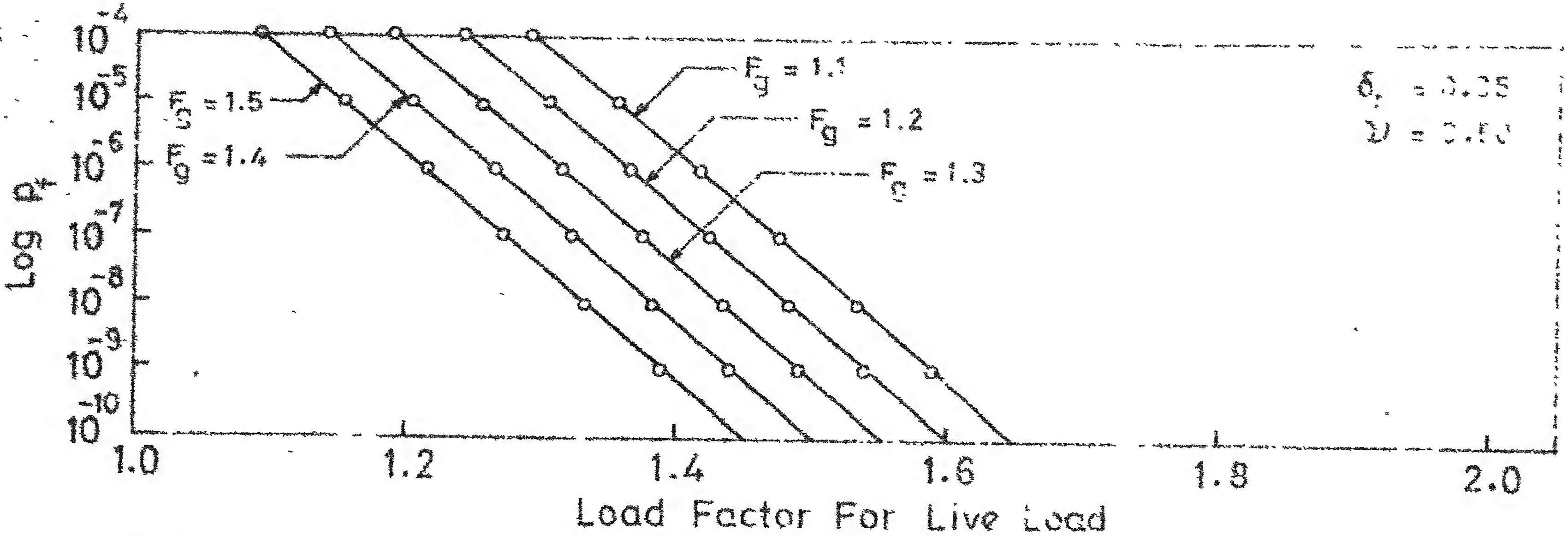


FIG. 7.4 VARIATION OF p_f AND F_q FOR δ_r = 0.05 AND ν = 0.5

For prestressed concrete buildings $F_g = 1.2$ and $F_q = 1.4$ are recommended.

At limit state of cracking, the coefficient of variation of the resisting moment of a PSC section is observed to be about 2.8 percent (Chapter 4) for probabilistic variation of σ_{cu} and σ_s. To be on the safer side, it has been taken as 3 percent for fixing the load factor. Taking $F_g = 1.1$ as per I.S. Code of practice on prestressed concrete (46) and fixing $p_f = 10^{-6}$, the value of F_q, using Eq. 7.13, is found to be 1.216 which is less than 1.25 specified by the I.S. Code.

7.6 CHARACTERISTIC AND DESIGN PRESTRESS

The characteristic value of the prestressing force, P_k, is defined by the relation

$$P_k = P_m (1 \pm k \ \delta_p) \tag{7.14}$$

where P_m is the mean value of the prestressing force and δ_p is the coefficient of variation of P arrising out of different sources of error.

The design value of the prestressing force, P_d, is given by

$$P_d = P_k \gamma_p \tag{7.15}$$

In these formulae, the positive sign (and as a corollary $\gamma_p > 1$) or the negative sign (and $\gamma_p < 1$) is chosen according to whether an increase or a decrease in prestress is detrimental. CEB-FIP Committee suggests $\gamma_p = 0.9$ for limit state of strength under maximum loading condition and $\gamma_p = 1$ for limit state of strength under minimum loading condition.

7.7 CHECK ON SAFETY

For the limit state under consideration the effects of design loads must be, at worst, equal to the absolute value permitted by the design strengths of the materials. The external bending moment caused by the ultimate load is a function of dead load (DL), live load (LL), load factors and geometry of the structure. The strength of the section is a function of strengths of materials, material reduction coefficients and geometric properties of the section. Hence the characteristic formula is written in the following manner for limit state of strength.

$$M_e(F_g \times DL,\ F_q \times LL) \leq M_r\left(\frac{\sigma_{cu}}{\gamma_c},\ \frac{\sigma_s}{\gamma_s},\ b_t,\ d,\ t_t,\ A_{ts}\right) \tag{7.16}$$

where γ_c and γ_s are material reduction factors for concrete and steel respectively. For statically determinate structures, $M_e(F_g \times DL)$, $M_e(F_q \times LL)$, etc. are equal to $F_g M_e(DL)$, $F_q M_e(LL)$, etc.

Hence Eq. 7.16 becomes

$$F_g M_g + F_q M_q \leq M_r\left(\frac{\sigma_{cu}}{\gamma_c},\ \frac{\sigma_s}{\gamma_s},\ b_t,\ d,\ t_t,\ A_{ts}\right) \tag{7.17}$$

Similar equations can be written for any other limit state. Knowing characteristic strengths of materials, material reduction factors, load factors etc., the semi-probabilistic design and analysis of PSC beams can be done as usual. This is illustrated with an example.

Example 7.1

A simply supported PSC beam of effective span 10m is subjected to a dead load of 1500 kg/m (including self weight of the beam) and a live load of 2000 kg/m.

Given :

$b_t = 50$ cm ; $h = 60$ cm;

$\sigma_{cum} = 422.8 \text{kg/cm}^2$; $s_o = 56 \text{ kg/cm}^2$

$\sigma_{sm} = 15680 \text{kg/cm}^2$; $s_s = 488 \text{kg/cm}^2$

$F_g = 1.2$; $F_q = 1.4$

The beam is designed based on limit state of strength at maximum loading (ultimate) at transfer of prestress.

Limit state of strength at maximum loading:

Top flange of the beam and area of steel are designed based on this limit state.

$$\text{ultimate load} = F_g \times DL + F_q \times LL$$

$$= 1.2 \times 1500 + 1.4 \times 2000$$

$$= 4600 \text{kg/m}$$

ultimate external bending moment is

$$M_{eu} = 4600 \times \frac{10^2}{8} = 57.5 \text{ tm}$$

Using Eq. 7.2, the characteristic strength of concrete,

$$\sigma_{cuk} = 422.8 - 1.64 \times 56 = 331 \text{ kg/cm}^2$$

Similarly, using Eq. 7.3, the characteristic strength of steel

$$\sigma_{sk} = 15680 - 1.64 \times 488 = 14880 \text{ kg/cm}^2$$

The design value of strength of concrete, σ_{cud}, is given by

$$\sigma_{cud} = \frac{\sigma_{cuk}}{\gamma_c}$$

using $\gamma_c = 1.5$

$$\sigma_{cud} = \frac{331}{1.5} = 220.7 \text{kg/cm}^2$$

Similarly, using $\gamma_s = 1.15$, the design value of strength of steel, σ_{sd}, is computed

$$\sigma_{sd} = \frac{14880}{1.15} = 13000 \text{ kg/cm}^2$$

The effective depth of beam is fixed as 54cms leaving 6cms from the bottom of the beam to the centroid of steel.

Assuming lever arm at failure as 0.95d i.e. $0.95 \times 54 = 51.3$ cms and the failure stress in steel is equal to its ultimate strength, the ultimate resisting moment of the section is given by

$$M_r = T_s \times \text{lever arm}$$

$$= T_s \times 51.3$$

From Eq. 7.15,

$$F_g M_g + F_q M_q \leq M_r$$

$$57.5 \times 10^2 \leq 51.3 \; T_s$$

Hence $$T_s \geq \frac{57.5 \times 10^2}{51.3} = 113.2 \text{ tonnes}$$

To equal the above value of T_s, the cross sectional area of concrete in compression must be

$$\frac{T_s}{\sigma_{cud}} = \frac{113.2 \times 10^3}{220.7} = 513 \text{ cm}^2$$

For neutral axis to lie inside the flange, approximate required thickness of flange at top is

$$t_t = \frac{513}{b_t} = \frac{513}{50} = 10.3 \text{ cm}$$

11 cm. is adopted.

It is now necessary to check the assumptions concerning M_r, failure stress in steel and the position of neutral axis.

The beam is designed as under-reinforced. Following I.S. specifications (46), the failure stress in steel is taken as equal to its ultimate strength for under-reinforced case. Assuming the neutral axis lies in the web (Fig. 7.5),

$$A_{tsf} = \frac{(b_t - b') \, t_t \, \sigma_{cud}}{\sigma_{sd}}$$

$$= \frac{(50-12) 11 \times 220.7}{13000} = 7.07 \text{ cm}^2$$

Resisting moment of overhanging portion of the flange, M_f, is

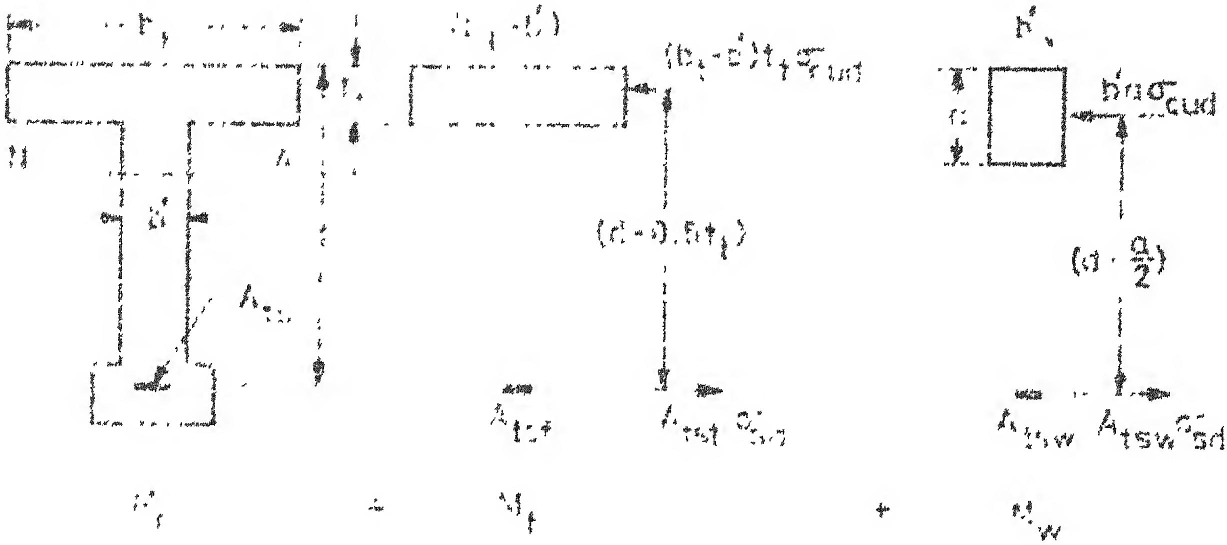
$$M_f = 7.07 \times 13000 \, (54 - 0.5 \times 11)$$
$$= 46 \text{ tm}$$

Moment to be resisted by web, M_w, is given by

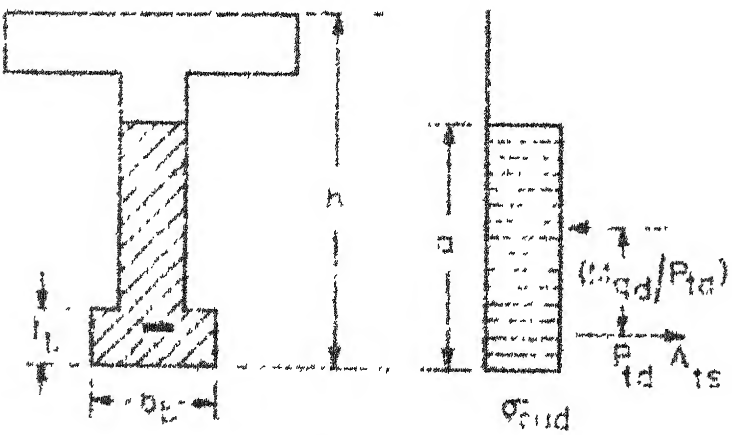
$$M_w = M_{eu} - M_f$$
$$= 57.5 - 46 = 11.5 \text{ tm}$$

Equation horizontal forces (Fig. 7.5),

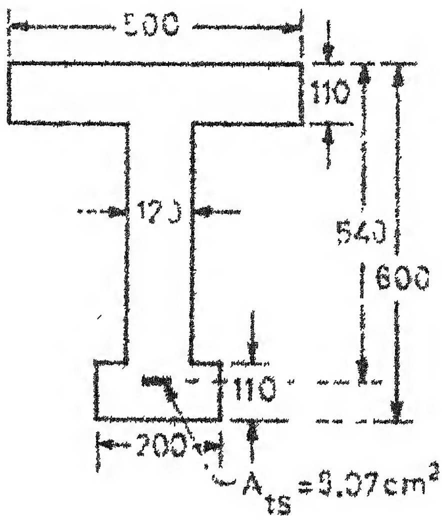
$$\sigma_{cud} \, b' a = \sigma_{sd} \, A_{tsw} \tag{7.18}$$



(a) DESIGN OF TOP FLANGE - LIMIT STATE OF STRENGTH AT MAXIMUM LOADING



(b) DESIGN OF BOTTOM FLANGE-LIMIT STATE OF STRENGTH AT TRANSFER OF PRE - STRESS



(c) SECTION

FIG. 7.5 EXAMPLE 7.1

DIMENSIONS ARE IN mm

Equating moments and using a reduction factor of 0.9,

$$0.9\ A_{tsw}\ \sigma_{sd}(d - \frac{a}{2}) = M_w \tag{7.19}$$

Substituting Eq. 7.18 in Eq. 7.19,

$$0.9\ A_{tsw}\sigma_{sd}\ d(1 - \frac{A_{tsw}\ \sigma_{sd}}{2b'd\sigma_{cud}}) = M_w$$

$$0.9\ A_{tsw} \times 13000 \times 54\ (1 - \frac{A_{tsw} \times 13000}{2 \times 12 \times 54 \times 220.7}) = 11.5 \times 10^5$$

Solving the above equation, the value of A_{tsw} is found to be $2 cm^2$.

$$\frac{A_{tsw}\ \sigma_{sd}}{b'd\sigma_{cud}} = \frac{2 \times 13000}{12 \times 54 \times 220.7} = 0.182 < 0.24$$

Hence the beam is under-reinforced and the failure stress in steel is equal to its ultimate strength. The value of area of steel is

$$A_{ts} = 7.07 + 2 = 9.07\ cm^2$$

Limit state of strength at transfer of prestress:

The bottom flange of the beam is designed based on this limit state.

As an increase in prestress is detrimental to the beam at the limit state of transfer of prestress, the characteristic strength of steel is given by

$$\sigma_{sk} = \sigma_{sm} + k\ s_s = 16480\ kg/cm^2$$

Taking the value of stress in steel at transfer as $0.8\sigma_s$, the characteristic prestressing force at transfer, P_{tk}, is

$$P_{tk} = 0.8\ \sigma_{sk}\ A_{ts}$$

$$= 0.8 \times 16480 \times 9.07 = 119500 \text{ kg}$$

The design value of initial prestressing force is given by

$$P_{td} = P_{tk}\ \gamma_s$$

using $\gamma_s = 1$, the value of P_{td} is 119500 kg. Decrease in dead load is detrimental to the beam at this limit state. Design dead load, M_{gd}, is taken as 0.9 times characteristic dead load. Hence

$$M_{gd} = 0.9 \times 1500 \times \frac{10^2}{8} = 16.875\text{tm}$$

Compression area required $= \dfrac{P_{td}}{\sigma_{cud}} = 543\text{cm}^2$

Assuming $(h-t_t) > a$ (Fig. 7.5b) and equating the distance of centroid of compression area from the centroid of steel to $\dfrac{M_g}{P_{td}}$, the following equation is obtained:

$$\frac{b'a(\frac{a}{2} - x_b) - (b_b-b')\ t_b(x_b-t_b \times 0.5)}{(\frac{P_{td}}{\sigma_{cud}})} = \frac{M_{gd}}{P_{td}}$$

or $$b'a(0.5a - x_b)-(b_b-b')t_b(x_b-0.5t_b) = \frac{M_{gd}}{\sigma_{cud}}.$$

Adopting $t_b = t_t = 11\text{cm}$,

$$12a(0.5a-6) - (b_b-12)12(6-5.5) = \frac{16.875 \times 10^5}{220.7}$$

Neglecting $(b_b-12)12(6-5.5)$, being small, the above equation is solved and the value of 'a' is found to be 42.25 cm. This is less than $(h-t_t)$ which is equal to 49cm. Hence the assumption $a < (h-t_t)$ is satisfied.

Equating the resisting capacity of the section to the prestressing force,

$$\sigma_{cud} \; [b'a + (b_b - b')t_b] = P_{td}$$

or

$$b'a + (b_b - b')t_b = \frac{P_{td}}{\sigma_{cud}}$$

$$12 \times 42.25 + (b_b - 12)11 = 543$$

or

$$b_b = 15.3\text{cm}$$

But on practical considerations, the bottom width of flange is adopted as 20cm . The designed section is shown in Fig. 7.5c.

The semi-probabilistic limit state design of a PSC beam has been illustrated for strength limit state and strength limit state at transfer of prestress. Similarly other limit states can also be considered and the design can be done. The semi-probabilistic limit state design method is suited for practical design purposes.

CHAPTER 8

DISCUSSIONS AND CONCLUSIONS

8.1 GENERAL

The results of the reliability analysis and design of PSC beams, presented in the previous chapters, are discussed herein. From the outcome of the results, conclusions are enumerated. Finally suggestions for further work are indicated.

8.2 RESULTS OF THE STATISTICAL ANALYSIS OF STRENGTH OF CONCRETE

The strength of concrete varies as a random phenomenon subjected to normal or lognormal distribution. The frequency distributions of the strengths of M350 and M250 concretes are normal with five percent level of significance. In the case of M200 concrete, where good supervision was provided while concreting, the distribution is normal with one percent level of significance. The distribution of the half of the groups of M150 concrete have satisfied the normal distribution with one percent level of significance while the other half groups have not. However,all the groups together have satisfied lognormal distribution with five percent level of significance. The skewness in M150 concrete is due to the fact that it is based on the nominal mix proportions with limited supervision. The concrete samples of M150 belong to many different projects and the quality of concrete is not of reliable nature in some of the projects. Considering all these aspects it can be concluded

that the strength of concrete follows normal distribution with atleast one percent level of significance it minimum control on the quality of the concrete is ensured.

Ninety one to ninety seven percent of M150, M200 and M 350 concrete samples satisfy the I.S. Code specifications. Only eighty percent of M250 concrete samples satisfy the Indian Code specifications. The mean value of the strength of M150 samples has been found to be much higher than the characteristic value; therefore most of the samples satisfied the code specifications even though variance of the strength is large.

One of the most important questions to be answered in the semi-probabilistic design is, what are the expected mean strength and variance of a concrete in the field, for a specified characteristic value. The three values can be connected though a linear relation as recommended by the CEB-FIP Committee as

$$\sigma_{cuk} = \sigma_{cum} (1 - k\delta)$$

in which the value of k assigns the probability of failure. The values of k for different groups of concrete vary from 1.21 to 2.01. The least-square fit of the field data of several groups of concrete indicates that the probability of failure of the concrete is 5.5 percent. The value of k corresponding to the above probability of failure is 1.6. This is close to the value of 1.64 (probability of failure is 5 percent) recommended by CEB-FIP Committee (45).

8.3 RESULTS OF THE STATISTICAL ANALYSIS OF STRENGTH OF STEEL

The strength of steel is found to be a random variable following normal distribution. The frequency distribution of strength of high tensile steel wires is normal with five percent level of significance. The proof strength of cold twisted deformed steel has also normal distribution with one percent significance level. The coefficient of variation of strength of 7 mm ϕ HTS wire and CTD steel are 3.1 and 4.7 percent respectively while it has been observed to be less than one percent in the case of 5 mm ϕ HTS wire. As the value of the ratio of mean strength of 5 mm ϕ HTS wire to its characteristic strength is high and the coefficient of variation is small, the probability of failure of the strength of 5 mm ϕ HTS is found to be less than 10^{-24}. The probability of failure of 7 mm ϕ HTS wire is found to be about 0.08. The probability of failure of CTD steel (diameter varying from 25 mm to 32 mm), based on yield strength, is 0.25.

8.4 RESULTS OF THE STATISTICAL ANALYSIS OF FLOOR LOAD IN OFFICE ROOMS

The frequency distribution of the floor load in office rooms is found to be lognormal with five percent significance level. The equivalent load intensity varies from 56 kg/m^2 to 293 kg/m^2. The coefficient of variation of load is observed to be about 0.4. The probability of load greater than the specified load depends on the characteristic load. For characteristic load of 400 kg/m^2, the probability of load greater than the above value is found to be 6×10^{-4}.

The mean value of the floor load is 130.77 kg/m^2. Even though this observed mean value of the floor load is small compared to the minimum loads specified by the I.S. Code for office buildings, the coefficient of variation of the observed load is high. Both parameters mean and coefficient of variation of the load are important as both have influence on the probability of failure of the structure. More extensive load data in similar office buildings is necessary to arrive at the characteristic values of the loads with certain confidence level so that they can be introduced into the code.

8.5 RESULTS OF THE STATISTICAL ANALYSIS OF GEOMETRIC PROPERTIES OF A SECTION

It has been observed that dimensions of a section are random variables with small coefficient of variation. All parameters of the section are normally distributed with atleast one percent level of significance. The coefficients of variation of width of flange, breadth of web and total depth of the section are negligible and significant variation has been observed in thickness of flange and effective depth of the beam.

8.6 RESULTS OF THE RELIABILITY ANALYSIS OF PSC BEAMS AT LIMIT STATE OF STRENGTH

It has been observed that even if a PSC section is under-reinforced based on deterministic analysis there is a probability of the section becoming over-reinforced. If probabilistic variations of σ_{cu} and σ_s are considered, the probability of the most critical sections

of the beams in Examples 3.1, 3.2 and 3.3 becoming over-reinforced is about 0.16, 0.10, and 0.124 respectively. The corresponding values of the ratios $(\frac{A_{tsw}\ \sigma_{sk}}{b'd\ \sigma_{cuk}})$ of the above sections are 0.234, 0.208 and 0.224 respectively. Hence, the probability of under-reinforced sections becoming over-reinforced increases with the increase in area of steel as expected. If the probabilistic variations of σ_{cu}, σ_s and dimensions of section are considered, the probability of the same sections becoming over-reinforced is about 0.07, 0.04 and 0.05 respectively which are less than the values if probabilistic variations of σ_{cu} and σ_s only are considered. This is because that mean values of the ratios $(\frac{A_{tsw}\sigma_{sk}}{b'd\sigma_{cuk}})$ are smaller as the mean values of A_{tsw}, b' and d are higher.

The probability of failure of a simply supported PSC beam (designed by working stress method as per I.S. Code) for deterministic load and probabilistic variations of σ_{cu} and σ_s is 1.08×10^{-7}. It is found that even though the probability of the section becoming over-reinforced is small, the probability of failure of the section depends on the section becoming over-reinforced and failure taking place. If probabilistic variations of σ_{cu} and σ_s and dimensions of section are considered, the probability of failure of the same beam is 1.4×10^{-7}. It can be concluded that the probability of failure of simply supported PSC beams (designed on working stress method)

at limit state of strength at design load is of the order of 10^{-7} for deterministic load.

For probabilistic load, the probability of failure of the simply supported PSC beam is 1.68×10^{-9} for probabilistic variations of σ_{cu} and σ_s and 1.84×10^{-10} for probabilistic variations of σ_{cu}, σ_s and dimensions of section. These failure probabilities are smaller than the values for deterministic load. This is because the mean value of the load is far less than the characteristic load when compared with the deterministic load even though coefficient of variation of the load is large. Here it is found that there are contributions to p_f values by under-reinforced and over-reinforced cases. Hence it can be said that the probability of the section becoming over-reinforced is also to be considered while computing failure probabilities of the beam at limit state of strength. It can be concluded that the probability of failure ofsimply supported PSC beam for probabilistic load is about 10^{-9}.

The probability of failure of continuous PSC beams shown in Figs. 3.12 and 3.13 for different cases are given in table 8.1. It can be said from the above table that probability of failure of continuous PSC.beams for deterministic load and for probabilistic variations of σ_{cu} and σ_s is of the order 10^{-17} and for probabilistic variations of σ_{cu} and σ_s and dimensions of section is of the order 10^{-16} .

Table 8.1. Probability of failure of continuous PSC beams at limit state of strength

	Bounds on Values of p_{fs} for	
	PV of σ_{cu} and σ_s	PV of σ_{cu}, σ_s and Dimensions of Section
Deterministic Load		
Two span beam (Fig. 3.12)	$6.17(-17)^* \leq p_{fs} \leq 12.57(-17)$	$2.65(-16) \leq p_{fs} \leq 6.16(-16)$
Three span beam (Fig. 3.13)	$1.80(-17) \leq p_{fs} \leq 5.01(-17)$	$1.10(-16) \leq p_{fs} \leq 2.42(-16)$
Probabilistic Load		
Two span beam (Fig. 3.12)	$3.22(-20) \leq p_{fs} \leq 6.46(-20)$	$3.82(-22) \leq p_{fs} \leq 7.66(-22)$
Three span beam (Fig. 3.13)	$3.40(-20) \leq p_{fs} \leq 9.78(-20)$	$5.00(-22) \leq p_{fs} \leq 14.2(-22)$

* 6.17(−17) is read as 6.17×10^{-17}.

For probabilistic load, the probability of failure of continuous beams for probabilistic variations of σ_{cu} and σ_s is of the order 10^{-20} and for probabilistic variations of σ_{cu}, σ_s and dimensions of sections is of the order 10^{-22}. The probability of failure of continuous beams is smaller than probability of failure of simply supported beams as expected.

8.7 RESULTS OF THE RELIABILITY ANALYSIS OF PSC BEAMS AT LIMIT STATE OF CRACKING

For deterministic load, the probability of cracking of the simply supported beam is 2.7×10^{-22} when probabilistic variations of σ_{cu} and σ_s are considered. When probabilistic variations of σ_{cu}, σ_s and dimensions of section are considered, it is observed that there is a slight increase in M_{rcm} but large increase in s_{rc} which results an increase in the value of p_c which is equal to 1.5×10^{-9}. For probabilistic load, the value of p_c of the simply supported beam for probabilistic variation of σ_{cu} and σ_s, is 2.94×10^{-6} and for probabilistic variation of σ_{cu}, σ_s and dimensions of section, is 4.87×10^{-7}.

The values of p_c for continuous beams are given in table 8.2. It is observed that for deterministic load the probability of cracking of continuous PSC beams varies from 10^{-6} to very small value less than 10^{-24} and for probabilistic load, it varies from

Table 8.2. Probability of cracking of continuous PSC beams

	Bounds on Values of p_{cs} for	
	PV of σ_{cu} and σ_s	PV of σ_{cu}, σ_s and Dimensions of Section
Deterministic Load		
Two span beam (Fig. 3.12)	1.6(-8)*	7.5(-6)
Three span beam (Fig. 3.13)	$2.08(-23) \le p_{cs} \le 4.16(-23)$	$6.00(-11) \le p_{cs} \le 12.94(-11)$
Probabilistic Load		
Two span beam (Fig. 3.12)	$3.98(-5) \le p_{cs} \le 4.03(-5)$	$7.02(-6) \le p_{cs} \le 7.11(-6)$
Three span beam (Fig. 3.13)	$4.15(-6) \le p_{cs} \le 12.41(-6)$	$7.31(-7) \le p_{cs} \le 21.87(-7)$

* 1.60(-8) is read as 1.60×10^{-8}.

10^{-5} to 10^{-7}. It is also observed that for probabilistic load, the probability of cracking of PSC beams is greater than the probability of failure of the same beams at limit state of strength.

8.8 RESULTS OF THE RELIABILITY ANALYSIS OF PSC BEAMS AT LIMIT STATE OF DEFLECTION

It is observed that the probability of the deflection exceeding the limit value at transfer of prestress is almost zero for simply supported and continuous PSC beams. This shows a large margin of safety.

For simply supported and continuous PSC beams, the probability of the deflection exceeding the limit value at design load is almost zero for deterministic load and probabilistic variations of σ_{cu} and σ_s. From table 8.3, it is observed that for probabilistic load, the value of p_{dd} for simply supported PSC beams varies from 7.27×10^{-10} to 1.81×10^{-12} and for continuous PSC beams from 10^{-11} to 10^{-12}. The values of p_{dd} for continuous PSC beams are smaller than the values of p_{dd} for simply supported PSC beams as expected. It is also seen that probability of failure of PSC beams at limit state of deflection is smaller than the values at limit state of strength and cracking.

8.9 RESULTS OF THE RELIABILITY ANALYSIS OF PSC BEAMS AT LIMIT STATE OF STRENGTH AT TRANSFER OF PRESTRESS

The probability of failure of the simply supported beam at limit state of strength at transfer of prestress is 5.32×10^{-4}

Table 8.3. Probability of deflections of continuous PSC beams exceeding their limit values under working load

	Bounds on Values of p_{dd}	
	Two Span Beam (Fig. 3.12)	Three Span Beam (Fig. 3.13)
PV of σ_{cu} and σ_s	0	0
PV of σ_{cu} , σ_s and Q	$2.88(-11)^* \leq p_{dd} \leq 5.76(-11)$	$3.40(-11) \leq p_{dd} \leq 3.8(-11)$
PV of σ_{cu}, σ_s , Q and dimensions of section	$0.70(-12) \leq p_{dd} \leq 1.39(-12)$	$1.63(-12) \leq p_{dd} \leq 1.65(-12)$

* 2.88(-11) is read as 2.88×10^{-11}.

for probabilistic variations of σ_{cu} and σ_s. When probabilistic variations of σ_{cu}, σ_s and dimensions of section are also considered, it is observed that mean value of κ representing the ratio of resistance to action, slightly increases and coefficient of variation of κ increases largely. These result increase in the value of p_t to 7.5×10^{-3} for the same beam.

The values of p_{ts} for continuous PSC beams are given in table 8.4. From the same table it can be said that when probabilistic variations of σ_{cu} and σ_s are considered, the value of p_{ts} of the beam is of the order 10^{-10} and when probabilistic variations of σ_{cu}, σ_s and dimensions of section are taken into account, it is of the order 10^{-8}. For continuous PSC beams the two possible failures are:

(i) the first yielding taking place at a section other than the end sections and the beam collapsing after the occurrence of a failure mode

(ii) the first yielding taking place at end sections and the beam reducing into overhang beam which automatically loads into collapse.

Both failures are to be investigated to determine the probability of failure of continuous PSC beam at limit state of strength at transfer of prestress.

Table 8.4. Probability of failure of continuous PSC beams at limit state of strength at transfer of prestress

	Values of p_{ts}	
	PV of σ_{cu} and σ_s	PV of σ_{cu}, σ_s and Dimensions of Section
Case 1 Failure (article 5.7)		
Two span beam (Fig. 3.12)	$3.40(-11)^* \leq p_{ts} \leq 6.80(-11)$	$2.13(-8) \leq p_{ts} \leq 4.26(-8)$
Three span beam (Fig. 3.13)	$1.40(-10) \leq p_{ts} \leq 2.08(-10)$	$7.39(-8) \leq p_{ts} \leq 14.78(-8)$
Case 2 Failure (article 5.7)		
Two span beam (Fig. 3.12)	5.00(-10)	2.95(-9)
Three span beam (Fig. 3.13)	4.10(-10)	3.30(-9)

* 3.4(-11) is read as 3.4×10^{-11}.

8.10 EFFECT OF p_f ON AREA OF STEEL FOR PROBABILISTIC LOAD

The effect of failure probability on area of steel and mean value of the ultimate resisting moment of the section has been studied in the case of probabilistic loads. The relation between p_f and M_{rm} and p_f and A_{ts} are shown in Figs. 8.1 and 8.2. This has been obtained for the simply supported PSC beam shown in Fig. 3.11 considering probabilistic variations of σ_{cu} and σ_s.

It is seen that an increase in p_f reduces the value of required resisting moment of the cross section and area of steel as anticipated. Same thing is observed when probabilistic variations of σ_{cu}, σ_s and geometric properties of the cross section are taken into account.

8.11 EFFECT OF FAILURE PROBABILITY ON LOAD FACTORS

Relation between probability of failure and load factors have been studied in Chapter 7 and are given in Figs. 7.1 to 7.4.

It is seen that the value of combined load factor increases with decrease in the value of probability of failure for constant coefficient of variation of resistance. It is also observed that the value of F_c increases with increase in δ_r for constant p_f.

It is observed from Fig. 7.2 to 7.4 that

(i) for constant values of ν and F_g, F_q increases if (a) p_f decreases and (b) δ_r increases for constant value of p_f

(ii) for constant values of δ_r and F_g, F_q increases if (a) p_f decreases and (b) ν increases for constant p_f

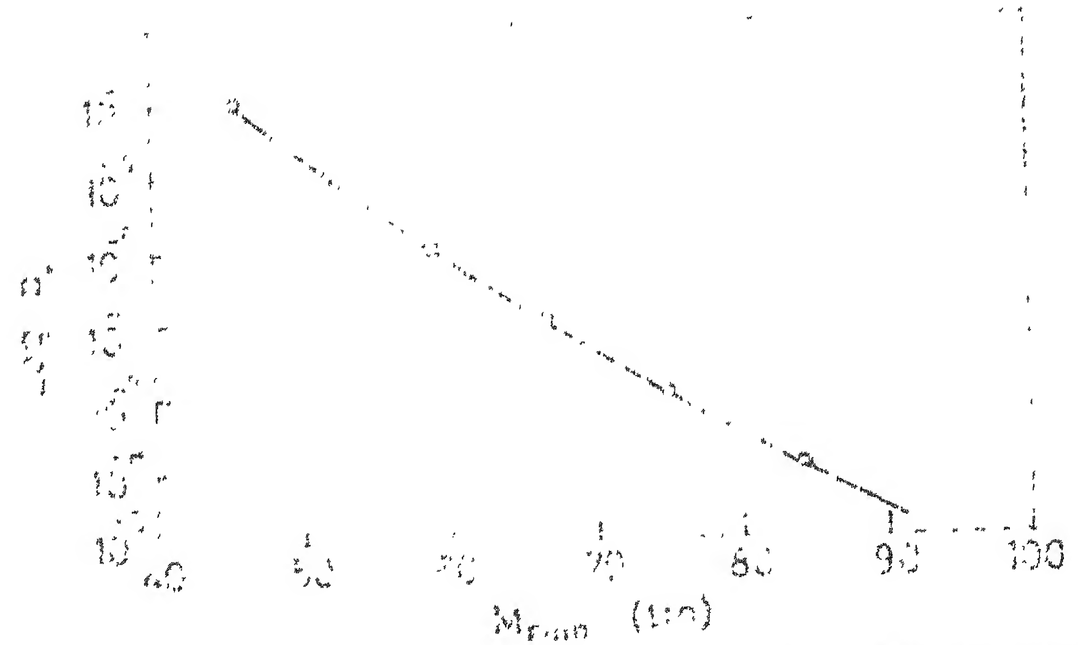


FIG. 8.1 EFFECT OF P_f ON ULTIMATE RESISTING MOMENT FOR PV OF Q, σ_{cu} AND σ_s

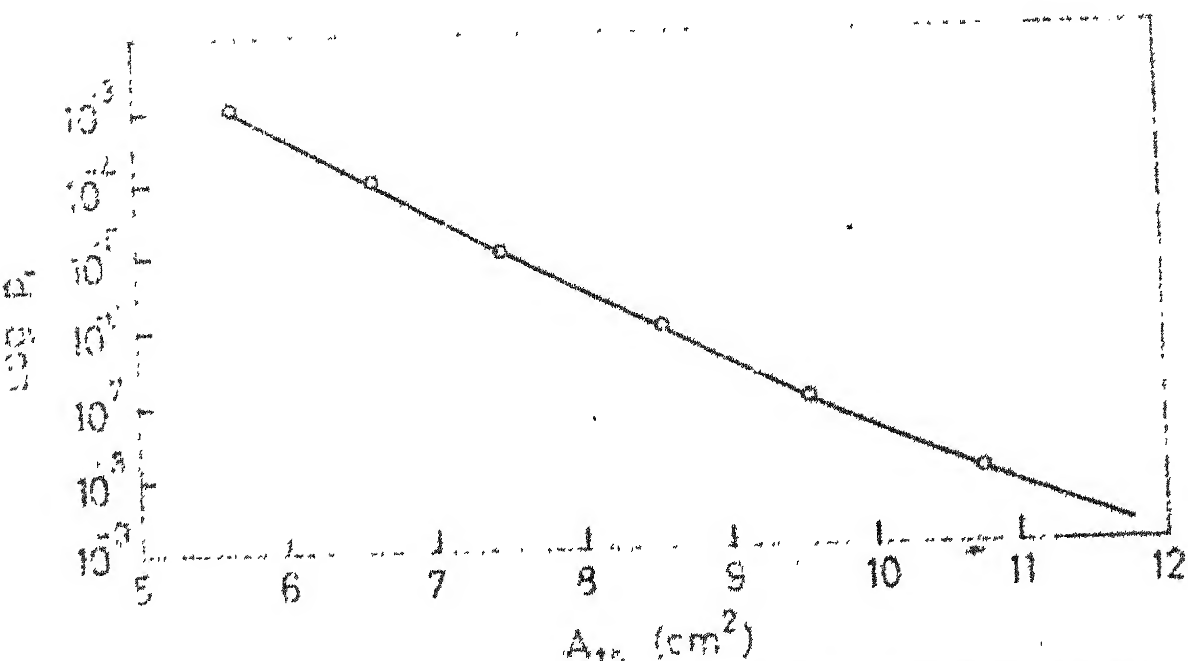


FIG. 8.2 EFFECT OF P_f ON AREA OF STEEL FOR PV OF Q, σ_{cu} AND σ_s

(iii) for constant values of δ_r and ν, F_q increases if (a) p_f decreases and (b) F_g decreases for constant p_f.

Coefficient of variation of ultimate resisting moment of PSC sections varies from 3 to 6 percent when probabilistic variations of σ_{cu} and σ_s are considered and from 5.3 to 7.6 percent when probabilistic variations of σ_{cu}, σ_s and geometric properties of the section are considered. If safety having probability of 10^{-6} is fixed, the value of F_q has been found to be 1.4 (Chapter 7) for $\delta_r = 0.05$ and $F_g = 1.2$. This value is close to the value recommended by Russian codes. Same values of $F_g=1.2$ and $F_q = 1.4$ are recommended for PSC building floors. These factors are based on the failure of the beam at limit state of strength.

The coefficient of variation of the resisting moment of a PSC section at limit state of cracking is observed to be about 2.8 percent. Taking its value of 3 percent and fixing p_c as 10^{-6}, the value of F_q has been found to be 1.22 for $F_g = 1.1$ and the ratio of dead load to live load equal to 0.75. The value of $F_q = 1.22$ is slightly less than 1.25 recommended by I.S. Code (46) for limit state of initial crack. The values of $F_g = 1.1$ to 1.2 and $F_q = 1.2$ to 1.3 are recommended for limit state of initial crack.

8.12 CONCLUSIONS AND RECOMMENDATIONS

1. The strength of concrete follows normal distribution with atleast one percent significance level if minimum control in the quality of concrete is ensured.

2. The most probable value of the failure of concrete cubes is 5.5 percent. The value of 1.64 is recommended for the characteristic coefficient which ensures a preassigned probability (5 percent) of the test results less than the characteristic strength.

3. The strength of high tensile steel follows normal distribution with 5 percent significance level.

4. The probability of failure of 7mm ϕ high tensile steel wire is 8.0 percent and of 5mm ϕ high tensile steel wire is less than 10^{-24} based on the characteristic values specified by the I.S. Code (46).

5. Floor load in office rooms follows lognormal distribution with five percent significance level. The mean value of the load is very much less than the characteristic load given by the I.S. Code (53); but its coefficient of variation is high.

6. Geometric properties of PSC sections follow normal distribution with at least one percent level of significance.

7. Even though a section is under-reinforced based on deterministic analysis, there is a probability of the same section becoming over-reinforced. This must be considered in computing probability of failure of PSC beams at limit state of strength at design load.

8. At limit state of strength, the probability of failure of (a) simply supported PSC beams is of the order 10^{-7} for

deterministic load and 10^{-9} for probabilistic load

(b) continuous PSC beams varies from 10^{-16} to 10^{-17} for deterministic load and from 10^{-20} to 10^{-22} for probabilistic load.

9. At limit state of cracking, the probability of cracking of

(a) simply supported PSC beams varies from 10^{-9} to 10^{-22} for deterministic load and from 10^{-6} to 10^{-7} for probabilistic load

(b) continuous PSC beams varies from 10^{-6} to very small value less than 10^{-24} for deterministic load and from 10^{-5} to 10^{-7} for probabilistic load .

It is observed that for probabilistic load, the probability of cracking of prestressed concrete beams is higher than the value at limit state of strength. This is because of the fact that for almost same combined standard deviation of (R-S) at both limit states, the difference between mean values of the resistance of the beams and action is small at cracking (that is overlap of distributions is more) compared to the large difference between mean values of resistance and action at limit state of strength. For deterministic load, the same conclusion is expected provided that the resistance of the section has same probability distribution and coefficient of variation at both limit state- cracking and strength. However, the relative probability of failure at limit states of strength and

cracking depends on many parameters such as

(i) coefficient of variation of resisting moment and external moment (ii) type of distribution functions for resisting moment and external moment and (iii) the level of intersection of both distributions. In general, the probability of cracking of prestressed concrete beams is higher than the probability of failure at limit state of strength.

10. At limit states of deflection at transfer of prestress, the probability of deflection in prestressed concrete beams exceeding the limit value is zero (less than 10^{-24}).

At limit state of deflection at design load, the probability of the deflection exceeding the limit value

(a) for simply supported and continuous prestressed concrete beams is zero (less than 10^{-24}) for deterministic load

(b) for simply supported prestressed concrete beams varies from 10^{-10} to 10^{-12} and continuous prestressed concrete beams from 10^{-11} to 10^{-12} for probabilistic load.

11. At limit state of strength at transfer of prestress, the probability of failure of simply supported prestressed concrete beams varies from 10^{-3} to 10^{-4} and continuous prestressed concrete beams from 10^{-8} to 10^{-10}.

12. Probability of failure at transfer condition of prestressed concrete beams is higher than that at limit states of strength

and deflection under working load condition. The probability of cracking of prestressed concrete beams is generally higher than the probability of failure at limit states of strength and deflection under working load condition.

8.13 SUGGESTIONS FOR FURTHER WORK

More data need be collected on the strength of steel so that the value of characteristic coefficient, which ensures the pressigned probability of the test results less than the characteristic strength, can be fixed with certain confidence level and recommended to I.S. Code. Extensive survey of floor loads in similar office buildings need be conducted to arrive at rational values of characteristic loads which can be included in the code.

Other limit states such as failure of anchorage zone (end blocks), buckling, torsion etc. are also to be considered in the reliability analysis and design of PSC beams.

As it is known that E_c, ε_c, ε_{ct}, σ_r and σ_{cu} and mutually independent, it is suggested that data may be collected on E_c, ε_c, ε_{ct} and σ_r and mutual dependence of the above variables may be established. Hence further research on the reliability analysis of prestressed concrete beams may be carried out taking into account of mutual dependence of the above variables and random variations of prestressing operations.

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APPENDIX

Table 1. Equivalent uniformly distributed load in office rooms in IIT, Kanpur

Sl. No.	Room No.	Office of	Area m^2	Total Load kg	Equivalent UDL kg/m^2
1	101	U.G. Academic	147.8	19321	131
2	102	Administrative	147.8	18140	123
3	103	Accounts	147.8	17038	115
4	104	Accounts	57.8	8049	139
5	105	Accounts	65.1	8218	126
6	208	Registrar	46.5	5456	117
7	205	Postgraduate	28	3656	131
8	210	Postgraduate	46.5	6576	141.5
9	211	Faculty administration	46.5	4756	102
10	204	Audit	28	2548	91
11	276	Dean of faculty	46.5	6276	135
12	212	Institute works department	69.7	13368	192
13	220	Estate	28	2329	83.3
14	219	Import section	46.5	7976	168
15	218	Director	46.5	7334	158
16	214	Director	28	1569	56
17	215	Director	28	3243	115.5
18	223	Dean of R & D	28	2435	87
19	275	Placement	46.5	4696	101

Sl. No.	Room No.	Office of	Area m^2	Total Load kg	Equivalent UDL kg/m^2
20	274	Standing Committee	46.5	8456	182
21	263	Elect.dept.	23.2	3388	146
22	313	Civil dept.	46.5	5415	116.5
23	332	Aero. dept.	46.5	6143	132
24	364	Mech. dept.	23.2	4888	210
25	381	Physics dept.	23.2	5896	254
26	412	Met. dept.	28	1868	67
27	432	Chemical dept.	46.5	5582	120
28	462	Chemical dept.	28	1942	69.5
29	461	Chemical dept.	14	2228	159
30	513	Maths. dept.	46.5	4189	90
31	567	Education development centre	23.25	2548	109.5
32	613	Humanities dept.	46.5	5062	109
33	WL 121	Dean of students	46.5	4896	105
34	WL 122	Asso. Dean of students	23.25	2022	87
35	273	Purchase	46.5	7364	158.5
36	221	Director	14	4101	293
37	201	Liason	23.25	1482	64
38	CS 101	Computer centre	20	3329	166.5